what can the tools of quantum information theory teach us about quantum magnets ?

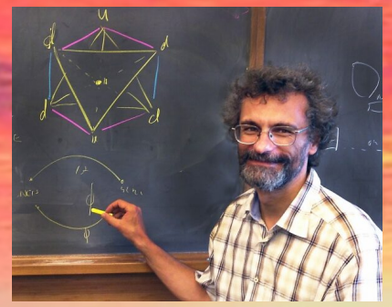
nic shannon





OIST

OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY
沖縄科学技術大学院大学





Quantum Spin Liquids 2025

Experimental Enigmas & Theoretical Challenges • 6–10 October 2025 • Budapest, Hungary

Welcome

Quantum spin liquids — highly entangled magnetic states that defy conventional order — stand at the frontier of modern condensed matter physics. Their topological excitations hold promise for fault-tolerant quantum technologies, and their realisation now spans natural crystals to programmable in-situ platforms such as Rydberg-atom quantum simulators. Breakthrough tools — from three-dimensional tensor-network techniques to machine-learning-assisted analytics — are rapidly evolving our view of these elusive phases.

This workshop brings together experimentalists and theorists working in the field of frustrated magnetism and quantum simulators. By uniting diverse expertise, we aim to develop new collaborations and push the boundaries of our understanding of quantum spin liquids.

We expect to host around 60 participants — approximately 50 international and 10 Hungarian researchers — including leading scientists as well as master's and doctoral students in condensed matter physics, materials science and quantum technology. The five-day program will feature invited lectures, contributed talks and a public lecture delivered by a distinguished guest, fostering an open exchange of ideas across all career stages.

We gratefully acknowledge the support of the Hungarian National Research, Development and Innovation Fund, the HUN-REN Wigner Research Centre for Physics, the Arrow ECS Kft., and IIT Madras, whose contributions have made this workshop possible. We also thank the HUN-REN Research Centre for Natural Sciences for hosting the event. As a contribution toward organizational expenses, we ask each participant for a registration fee of 300 EUR.

Supported by

National Research, Development and Innovation Office Hungary

Program Financed from the NRDI Fund

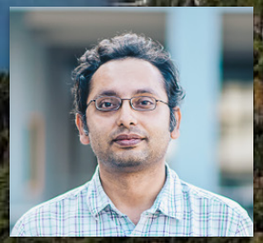
HUN-REN

Wigner

arrow

IBM Distributor

Indian Institute of Technology Madras







The Abdus Salam International Centre for Theoretical Physics

IAEA

unesco

Advanced School and Conference on Quantum Matter

Description:

This school and conference will feature pedagogical school lectures and conference style talks on Quantum Spin Liquids in an intertwined manner over two weeks. Our aim is to communicate the exciting developments to students and young researchers as well as present state of the art in the field.

MORE INFORMATION:

We are currently witnessing rapid developments concerning the realization of quantum spin liquids in programmable quantum simulators and via Floquet engineering. Similarly, the realization of exotic fractonic phenomena in quantum spin liquids described by higher-rank gauge theories has now become possible in spin models with pursuits for their search in quantum materials. Over the past few years, we have achieved a deepened understanding of the problem of symmetry classification of quantum spin liquids relying on ideas and methods from category theory, and advanced notions in group theory. Substantial progress has also been made to understand the stability of gapless quantum spin liquids. Simultaneously, many-body numerical approaches based on neural network constructions, tensor network methods, and machine-learning approaches have allowed one to shed light on exploring quantum state preparation and its evolution.

TOPICS:

- General introduction to Quantum Spin Liquids - Fractionalization and Topological Order, and types of Phases
- Partons and Gauge Theory - Symmetry Classification schemes
- Fusion rules, Categories and QSL classification
- Fractons and Higher-Rank Gauge Theories
- Microscopic Models and Materials
- Theoretical and Numerical Approaches - Machine learning methods, neural network constructions
- Quantum State Preparation

SCHOOL LECTURERS:

L. Balents, Kavli Institute for Theoretical Physics, Santa Barbara, USA
F. Becca, University of Trieste, Italy
S. Gazit, The Hebrew university of Jerusalem, Israel
M. Hermele, University of Colorado Boulder, USA
H-Y. Kee, University of Toronto, Canada
Y.-M. Lu, The Ohio State University, USA
R. Samajdar, Princeton University, USA
S. Trebst, University of Cologne, Germany
J. Vidal, Sorbonne University Paris, France

CONFERENCE SPEAKERS:

F. Assaad, University of Würzburg, Germany
E. Berg, Weizmann Institute of Science, Israel
P. Bonetti, Harvard University, USA
S. Capponi, University of Toulouse, France
G. Chen, Peking University Beijing, China
G. Y. Cho, Korea Advanced Institute of Science & Technology, South Korea
A. Chubukov, University of Minnesota, USA
C. Liu, University of California Berkeley, USA
B. Doucot, Sorbonne University Paris, France
Y.-C. He, Stony Brook University, USA
Y.-B. Kim, University of Toronto, Canada
J. Knolle, Technische Universität München, Germany
A. Läuchli, EPFL and PSI, Switzerland
S.-B. Lee, Korea Advanced Institute of Science & Technology, South Korea
Z. Yang Meng, The University of Hong Kong, Hong Kong
L. Messio, Sorbonne University Paris, France
R. Moessner, MPIPKS Dresden, Germany
E.-G. Moon, Korea Advanced Institute of Science & Technology, South Korea
J. Moore, University of California Berkeley, USA
Y. Motome, University of Tokyo, Japan
M. Oshikawa, University of Tokyo, Japan
A. Paramekanti, University of Toronto, Canada
K. Penc, HUN-REN Wigner Research Centre For Physics Budapest, Hungary
N. Perkins, University of Minnesota, USA
H. Pichler, University of Innsbruck, Austria
F. Pollmann, Technische Universität München, Germany
Y. Ran, Boston College, USA
A. Sandvik, Boston University, USA
N. Shannon, Okinawa Institute of Science and Technology, Japan
K. Shtengel, University of California Riverside, USA
I. Sodemann, University of Leipzig, Germany
V. Tripathi, Tata Institute of Fundamental Research Mumbai, India
J. Von Delft, Ludwig-Maximilians-University, München, Germany
C. Wang, Perimeter Institute for Theoretical Physics, Canada
A. Wietek, MPIPKS Dresden, Germany
C. Xu, University of California, Santa Barbara, USA
Y. Zhou, Chinese Academy of Sciences, China
L. Zou, National University of Singapore, Singapore

1 - 12 December 2025

Trieste, Italy

Application and Deadlines:

For applicants requesting financial and/or visa support:
1 August 2025

For all other applicants:
1 October 2025

DIRECTORS:

S. Bhattacharjee
International Centre For Theoretical Sciences, India
Y. Iqbal
Indian Institute of Technology Madras, India
S. Sachdev
Harvard University, USA

LOCAL ORGANISER:

M. Kiselev, ICTP, Italy

FURTHER INFORMATION:

E-mail: smr4114@ictp.it
Web: http://indico.ictp.it/event/10879/

Female scientists are encouraged to apply.

GRANTS:

A limited number of grants are available to support the attendance of selected participants, with priority given to participants from developing countries.
There is no registration fee.

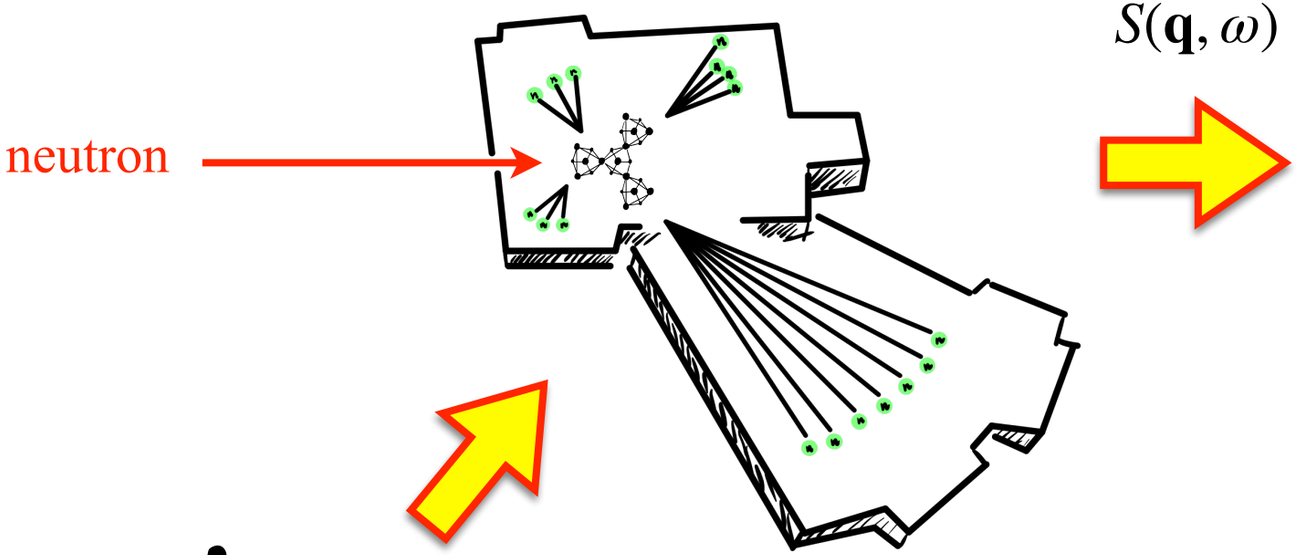


quantum spin liquids

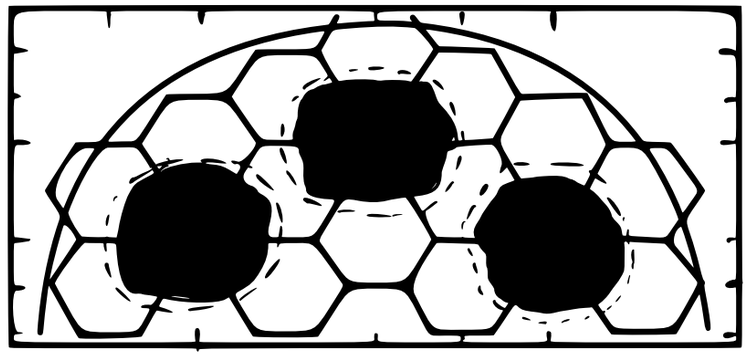


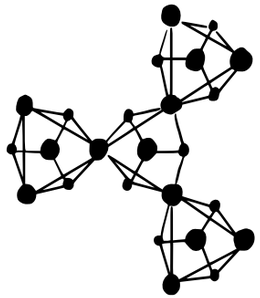
quintessential "quantum" of phases of matter
with long-range entanglement, and fractionalized excitations

groundhog day, QSL style...



$S(\mathbf{q}, \omega)$





quantum
spin liquid
candidate

no magnetic
Bragg-Peaks $\Rightarrow$ **not** magenticaly
ordered state

discovery paper: convincing fit to diffuse scattering, coming from QSL

rebuttal: equally convincing fit to diffuse scattering from state driven by disorder

the very things which make QSL interesting, make them difficult to distinguish in experiment



$YbMgGaO_4$





nature physics
LETTERS
PUBLISHED ONLINE: 5 DECEMBER 2016 | DOI: 10.1038/NPHYS3971

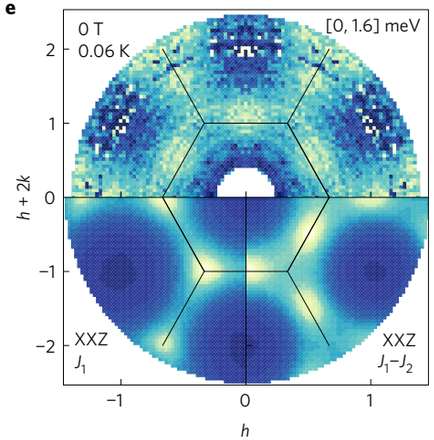
Continuous excitations of the triangular-lattice quantum spin liquid $YbMgGaO_4$

Joseph A. M. Paddison[1], Marcus Daum[1†], Zhiling Dun[2†], Georg Ehlers[3], Yaohua Liu[3], Matthew B. Stone[3], Haidong Zhou[2] and Martin Mourigal[1*]

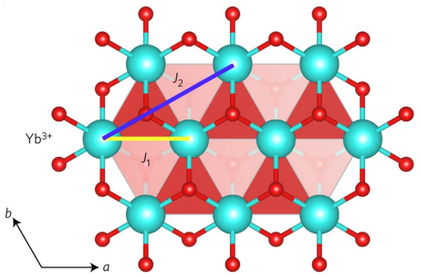
PRL 119, 157201 (2017) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2017

Disorder-Induced Mimicry of a Spin Liquid in $YbMgGaO_4$

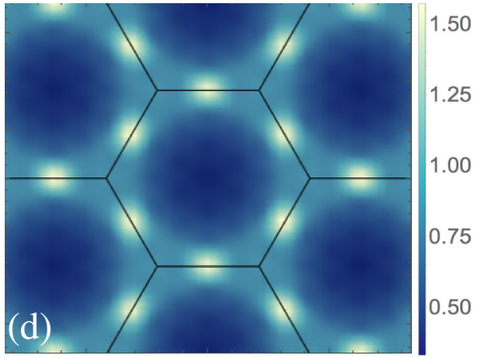
Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev
Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
(Received 8 March 2017; published 11 October 2017)



spin correlations explained in terms of quantum fluctuations



triangular-lattice AF based on Yb^{3+}



spin correlations explained in terms of effect of disorder

...is there a more reliable way of identifying QSL in systems with disorder ?

what makes quantum fluctuations special in the first place...?



...entanglement





PHYSICAL REVIEW B **103**, 224434 (2021)

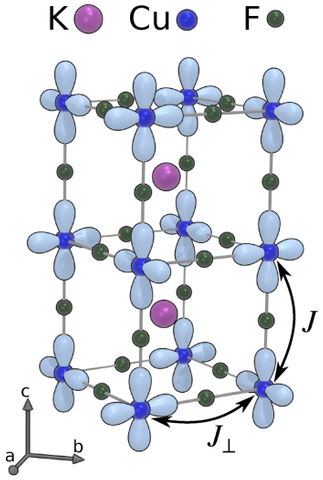
Editors' Suggestion

Witnessing entanglement in quantum magnets using neutron scattering

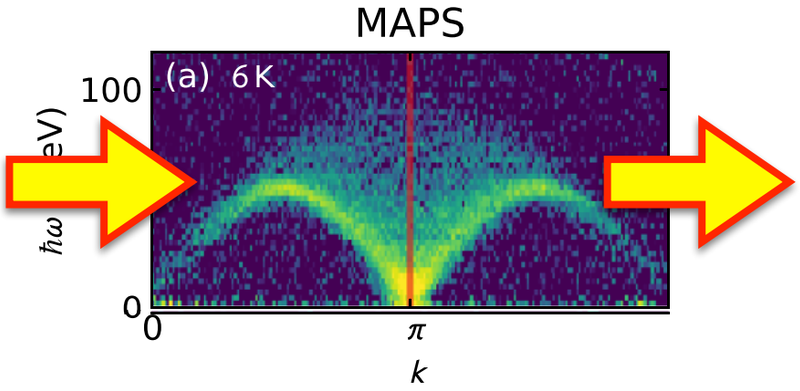
A. Scheie,[1,*,†] Pontus Laurell,[2,3,*,‡] A. M. Samarakoon,[1] B. Lake,[4,5] S. E. Nagler,[1,6] G. E. Granroth,[1] S. Okamoto,[7,6] G. Alvarez,[2,3] and D. A. Tennant[1,6,8,§]

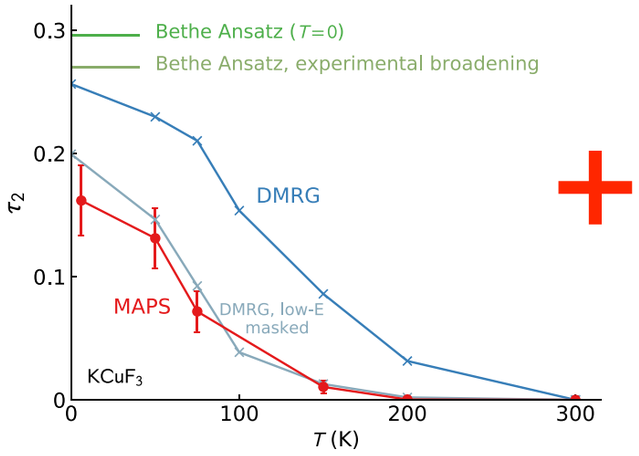
erratum: A. Scheie *et al*, Phys. Rev. B **107**, 059902(E) (2023)

$KCuF_3$

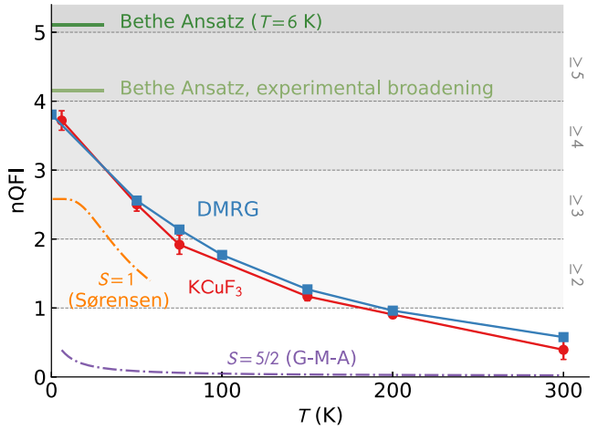


s=1/2
Heisenberg
spin chain





measure of entanglement
of pairs of spins

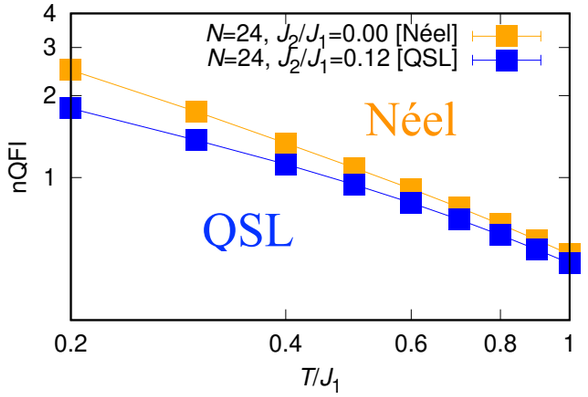


measure of entanglement
of many spins

promising new approach to characterizing entanglement in magnets...

question for today.....

can experimentally-accessible measures of entanglement distinguish quantum spin liquid and disorder-driven states ?



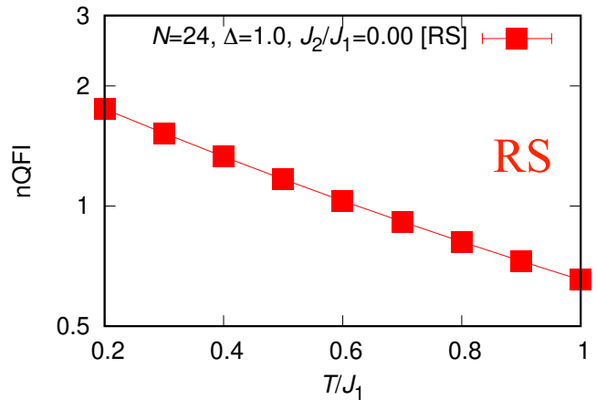
power law:
$nQFI \sim T^{-c}$

...yes, at least in some cases



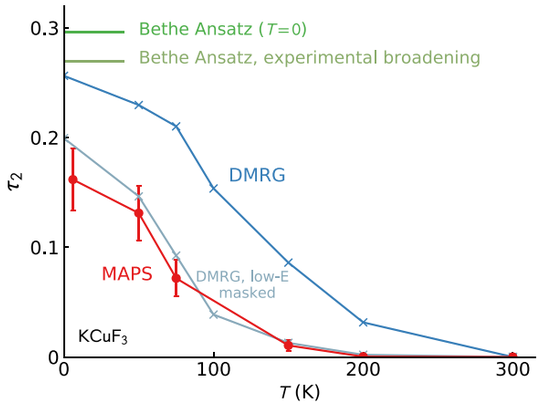
(but there are plenty of traps to fall into !)





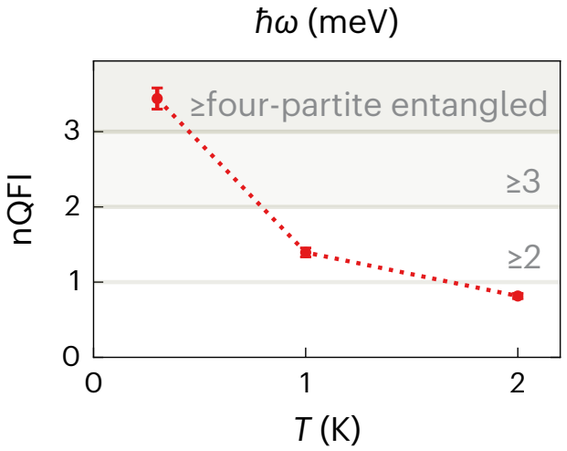
exponential:
$nQFI \sim \exp[-c'T]$

what will we learn along the way ?



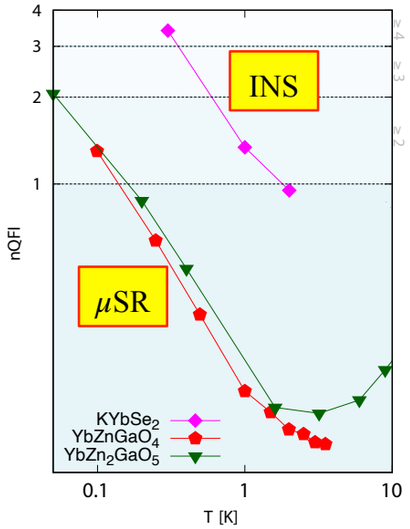
why do experimental results for 2-tangle in $KCuF_3$ differ from simulations ?

concurrence is an unreliable witness in presence of disorder



is it safe to rule out random singlet phases on the grounds of multipartite entanglement ?

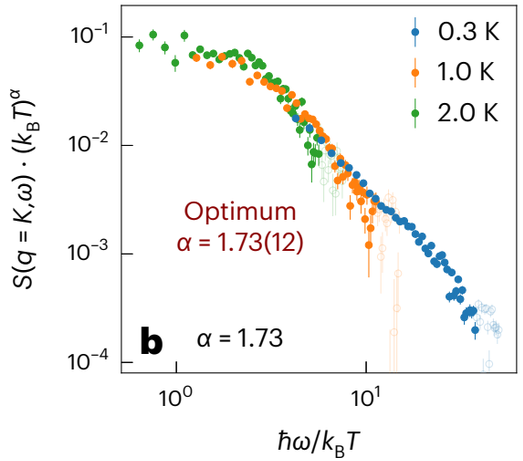
no



how should we interpret the power-law scaling of QFI seen in triangular lattice AF's ?

characteristic of low-T growth of entanglement in QSL and Néel phases

if there's time...



can we understand the way in which fluctuations scale with energy and temperature in the triangular lattice AF $KYbSe_2$?

consistent with proximity to quantum critical point

wouldn't have happened without...



Snigdh Sabharwal
(OIST)



Tokuro Shimokawa
(OIST)

for a long time,
quantum spin
liquids were
considered
to be mythical
creatures...



...that changed !

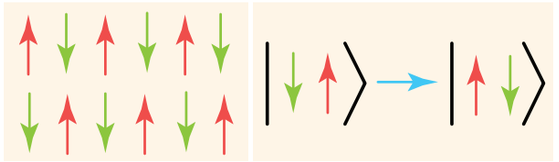
PHYSICS

An End to the Drought of Quantum Spin Liquids

Patrick A. Lee

After decades of searching, several promising examples of a new quantum state of matter have now emerged.

Electrons possess magnetic behavior through the quantum mechanical property of spin. The magnetic properties of materials then arise from the collective interaction of electrons on atoms within the crystal. Below a transition temperature, the electron spins of normal magnets "freeze" into an ordered array of magnetic dipoles. Whether the ordering is ferromagnetic (all the dipoles point in the same direction) or antiferromagnetic (the

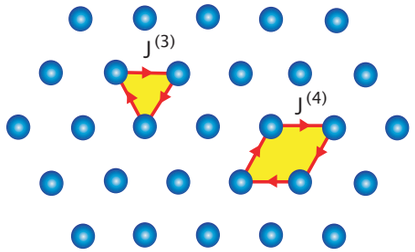


Ordered spins. (**Left**) Néel's picture of antiferromagnet ordering with an alternate spin-up–spin-down pattern across the lattice. (**Right**) Quantum fluctuations lead to mutual spin flips, which Landau argued would disorder Néel's state.

Patrick A. Lee, Science **321**, 1306 (2008)

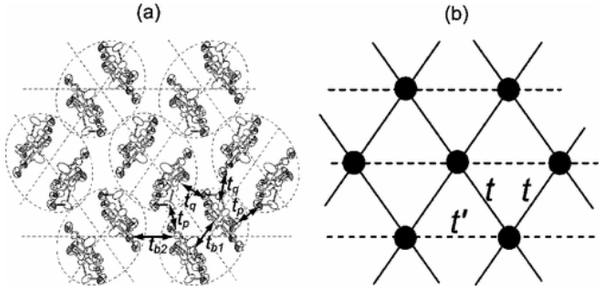
QSL candidates are frequently controversial...

^{3}He on graphite



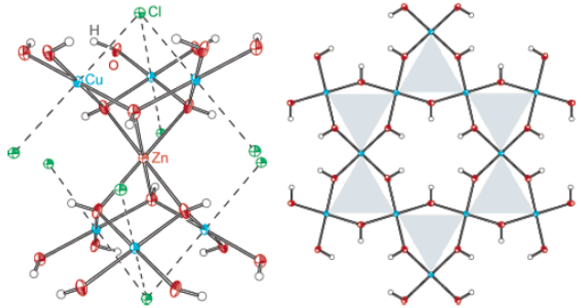
K. Ishida *et al.*,
PRL **79**, 3451 (1997)

κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$



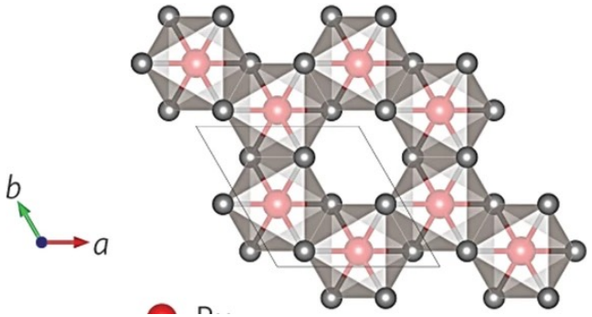
Y. Shimizu *et al.*
PRL **91**, 107001 (2003)

Herbertsmithite



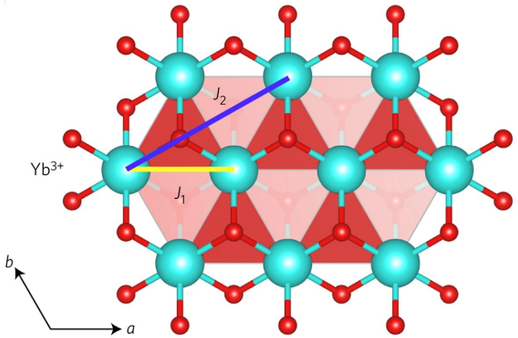
M.P. Shores *et al.*
JACS **127**, 13462 (2005)

α-RuCl$_3$



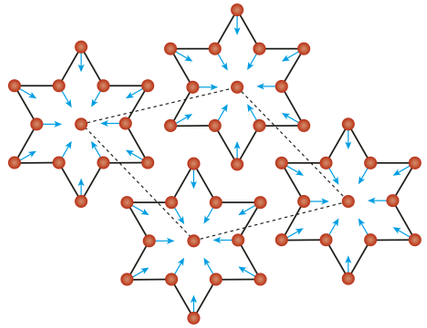
A. Banerjee *et al.*,
Nat. Mat. **15**, 733 (2016)

YbMgGaO$_4$



J. A. M. Paddison *et al*,
Nat. Phys. **13**, 117 (2017)

1T-TaS$_2$



K. T. Law *et al*,
PNAS **114**, 6996 (2017)

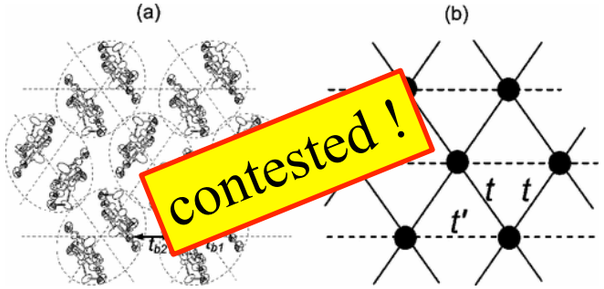
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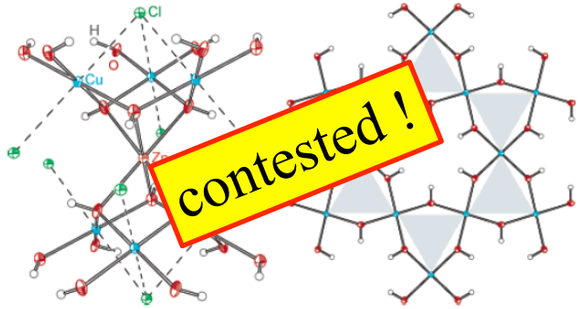
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κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$



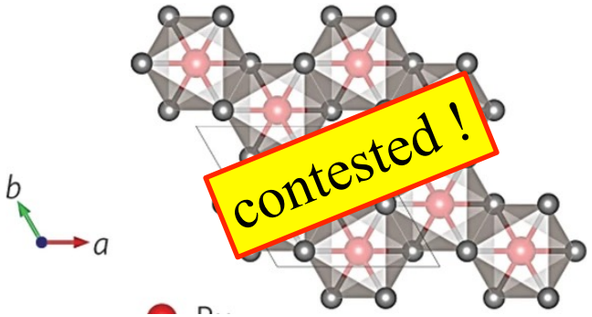
Y. Shimizu *et al.*
PRL **91**, 107001 (2003)

Herbertsmithite



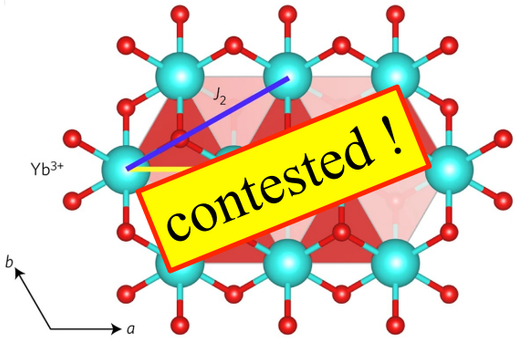
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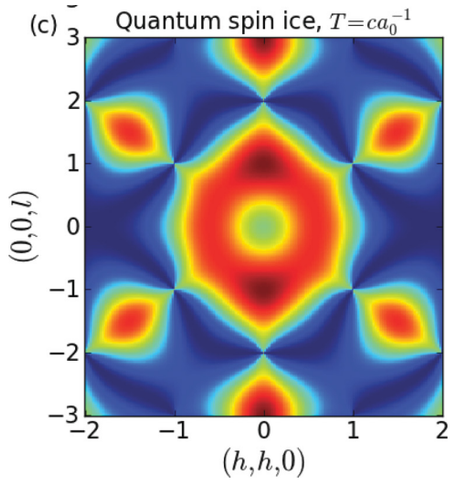


K. T. Law *et al*,
PNAS **114**, 6996 (2017)

why are spin liquids difficult to interpret ?

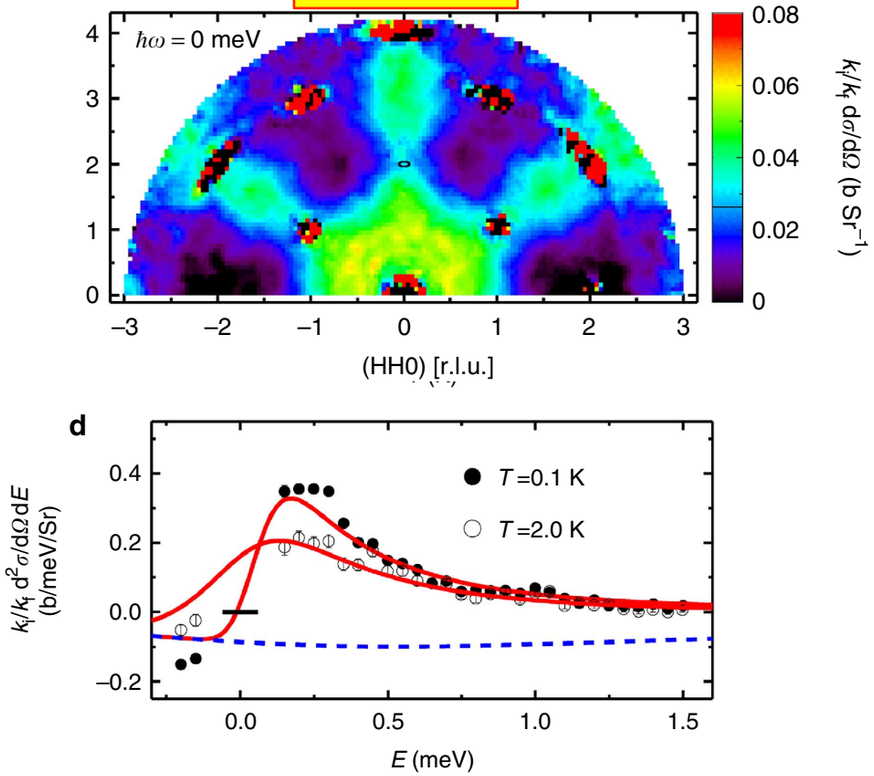
structured diffuse scattering can have many origins, including disorder

prediction for quantum spin ice (QSL)



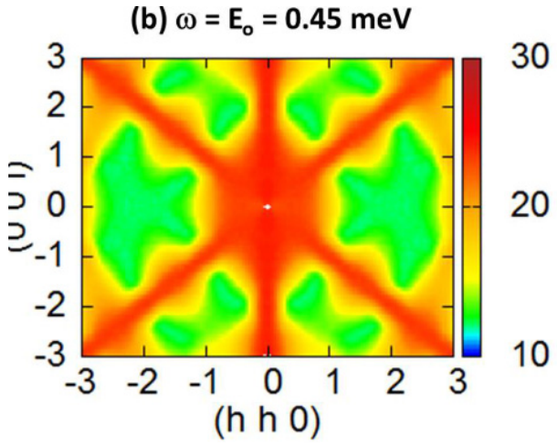
O. Benton *et al.*,
Phys. Rev. B **86**, 075154 (2012)

$Pr_2Zr_2O_7$



K. Kimura *et al.*,
Nat Commun **4**, 1934 (2015)

interpretation in terms quadrupolar order



S. Petit *et al.*,
Phys. Rev. B **94**, 165153 (2016)

modern papers on QSL routinely invoke "entanglement"...

IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. **80** (2017) 016502 (55pp)

doi:10.1088/0034-4885/80/1/016502

Review

Quantum spin liquids: a review

Lucile Savary[1] **and Leon Balents**[2]

hard to agree on a *positive* definition of a QSL. Here, we will take the point of view that the essential ingredient of a QSL is not the lack of order, but the presence of an anomalously high degree of entanglement, or massive quantum superposition. QSLs are exemplars of highly entangled quantum matter in the context of magnetism.

nature physics

LETTERS

https://doi.org/10.1038/s41567-018-0116-x

In a quantum spin liquid, the magnetic moments of the constituent electron spins evade classical long-range order to form an exotic state that is quantum entangled and coherent over macroscopic length scales[1,2]. Such phases offer promis-

Experimental signatures of emergent quantum electrodynamics in $Pr_2Hf_2O_7$

Romain Sibille[1,2]*, Nicolas Gauthier[2], Han Yan[3], Monica Ciomaga Hatnean[4], Jacques Ollivier[5], Barry Winn[6], Uwe Filges[2], Geetha Balakrishnan[4], Michel Kenzelmann[1,2], Nic Shannon[3] and Tom Fennell[1]



DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS

By E. SCHRÖDINGER

[Communicated by Mr M. Born]

[*Received* 14 August, *read* 28 October 1935]

introduces term "*entanglement*"



...written in response to the EPR paper (March 1935)

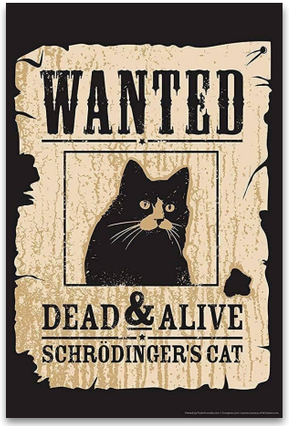
"The best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts."

E. Schrödinger, Math. Proc. Camb. **31**, 555 (1935)

$$|\Psi_{AB}\rangle = \sum_{n,m} c_{nm} |n\rangle_A \otimes |m\rangle_B \neq |\Psi_A\rangle \otimes |\Psi_B\rangle \quad c_{nm} \geq 0$$

quantum state of system A ∪ B (complete knowledge within QM)

not (generally) possible to write as product of states of A and B



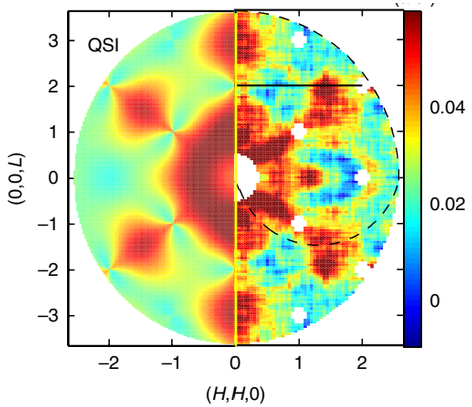
"I would not call that one but rather the characteristic trait of quantum mechanics [...] the two ψ-functions have become entangled."

E. Schrödinger, Naturwissenschaften **23**, 807 (1935)

introduces term „*Verschränkung*". And his cat.

how many spins can we entangle ?

$Pr_2Hf_2O_7$



(emergent) speed of light

$$c \approx 0.3 \text{ meV Å} \approx 4 \text{ ms}^{-1}$$

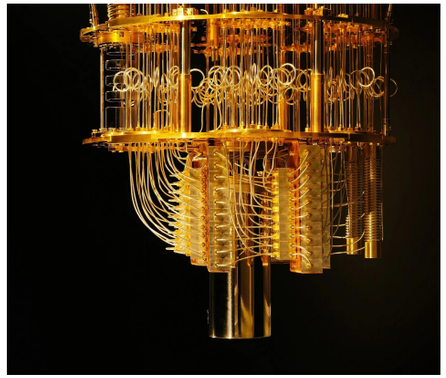
Sibille *et al.,* Nat. Phys. **14**, 711 (2018)

thermal de Broglie wavelength of emergent photons @ 50 mK

$$\lambda_c = \frac{\pi c}{T} \approx 240 \text{ Å}$$

each spin entangled with ~10^6 other spins

IBM Eagle



127 qubits @ 10 mK

$Pr_2Hf_2O_7$ @ 10 mK

$\lambda_c \approx 1200$ Å

each spin entangled with ~10^8 other spins

😃

how can we quantify entanglement ?

von Neuman's definition of entropy:

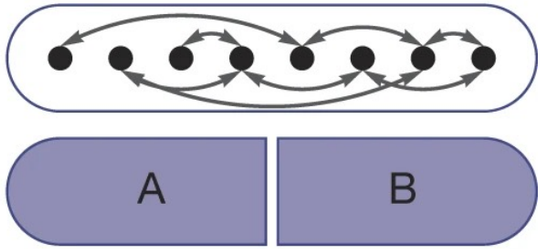
$$\mathcal{S}(\rho) = -\,\mathrm{Tr}\left[\rho \log \rho\right]$$

"**entanglement entropy**" associated with a bipartition of the system

$$\mathcal{S}(\rho_A) = -\,\mathrm{Tr}\left[\rho_\mathrm{A} \log \rho_\mathrm{A}\right]$$

reduced density matrix: $\rho_A = \mathrm{Tr}_\mathrm{B}(\rho_\mathrm{AB})$

C.H. Bennett *et al.*, Phys. Rev. A **53**, 2046 (1996)



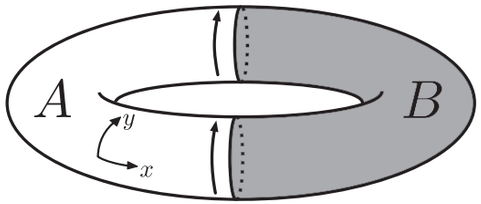
$$|\psi\rangle \neq |\psi\rangle_\mathrm{A} \otimes |\psi\rangle_\mathrm{B}$$

caveat emptor

interpretation in terms of entanglement valid only for pure states at T=0

reduced density matrix cannot be measured in experiment (*)

useful theoretical tool for characterizing gapped, topological phases at T=0



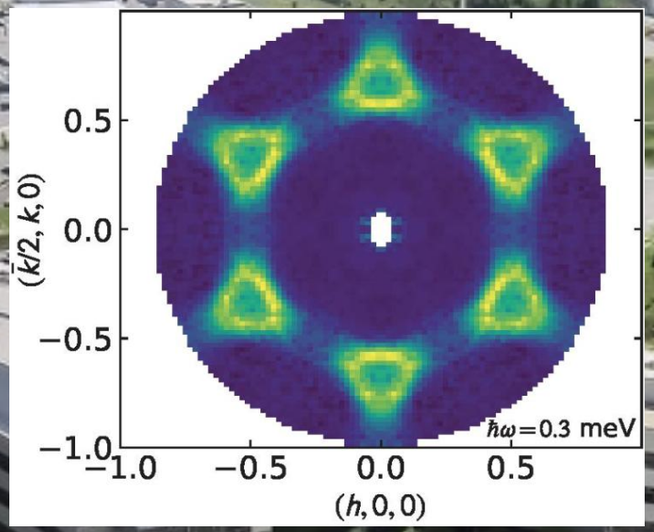
$$S = (\alpha + \log 2)L - \log 2$$

area law topological term

A. Kitaev and J. Preskill, Phys. Rev. Lett **96**, 110404 (2006)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006)

(*) possible for cold atoms

Spallation Neutron Source (SNS), Oak Ridge, TN



$1bn $\Rightarrow S(\mathbf{q}, \omega, T) \nRightarrow S(\rho_A)$

does this mean that quantum information ideas are useless, so far as real materials are concerned ?

how else can we explore entanglement ?

in the late 1990's the quantum information community became interested in designing "tests" for entanglement



Separability of mixed states: necessary and sufficient conditions

Michał Horodecki [a], Paweł Horodecki [b], Ryszard Horodecki [c]

[a] *Department of Mathematics and Physics, University of Gdańsk, 80-952 Gdańsk, Poland*

[b] *Faculty of Applied Physics and Mathematics, Technical University of Gdańsk, 80-952 Gdańsk, Poland*

[c] *Institute of Theoretical Physics and Astrophysics, University of Gdańsk, 80-952 Gdańsk, Poland*

M. Horodeki, P. Horodecki and R. Horodecki, Phys. Lett. A **223**, 1 (1996)

N.B. ability to handle mixed states is necessary for any measure at finite temperature (cf. experiment)



Bell inequalities and the separability criterion

Barbara M. Terhal

IBM Research Division, Yorktown Heights, NY 10598, USA

B. Terhal, Phys. Lett. A **271**, 319 (2000)

"We show that a violation of a Bell inequality can formally be expressed as a witness for entanglement."

Review of "Entanglement Detection": O. Gühne and G. Tóth, Physics Reports **474**, 1–75 (2009)

many objects can serve as entanglement witnesses...

REVIEW

ADVANCED QUANTUM TECHNOLOGIES

www.advquantumtech.com

Witnessing Entanglement and Quantum Correlations in Condensed Matter: A Review

Pontus Laurell, Allen Scheie, Elbio Dagotto, and D. Alan Tennant**

...a surprising number of which are already familiar !

Witness	Key Formulae	Physical Interpretation	Experimental Probes
Susceptibility (χ_{EW})	Equation (16)	generalized spin-squeezing	χ_m
One-tangle (τ_1)	Equation (26)	one-site entanglement	diffraction
Two-tangle (τ_2)	Equation (53)	total pairwise entanglement	$S(\mathbf{Q})$
Concurrence (C)	Equations (47, 52)	pairwise entanglement	$S(\mathbf{Q})$, C_v, χ_m
Quantum discord [$\mathcal{Q}(\rho)$]	Equations (60, 105, 106)	pairwise quantum correlations	$S(\mathbf{Q})$, C_v, χ_m
Quantum Fisher Information (QFI)	Equations (108, 111)	multi-partite entanglement	spectroscopy
Quantum variance (QV)	Equation (129)	multi-partite entanglement	spectroscopy
Skew information (SI)	Equations (129, 130)	multi-partite entanglement	spectroscopy
Spatial quantum correlations	Equation (131)	quantum fluctuations	spectroscopy

P. Laurell *et al.*, Advanced Quantum Technologies **8**, 2400196 (2025)

many objects can serve as entanglement witnesses...

REVIEW

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$S(\mathbf{q}, \omega)$

measured by INS

P. Laurell *et al.*, Advanced Quantum Technologies **8**, 2400196 (2025)

review of application to inelastic neutron scattering: A. Scheie *et al.*, Materials Today Quantum **5**, 100020 (2025)

so what is concurrence ?

concurence quantifies **entanglement of formation** for an arbitary quantum state of two qubits (spins)
(i.e. the entanglement needed to create that state)

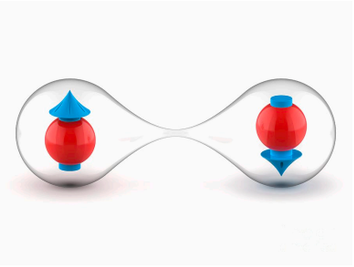


$$0 \le C(\rho) \le 1$$

separable state
(no entanglement)

density matrix
for pair qubits

Bell state, e.g. singlet
(maximially entangled)



concurrence can be calculated as

$$C(\rho) = \max\{0,\ \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \qquad \lambda_1 \ge \lambda_2 \ge \lambda_3 \ge \lambda_4$$

$$\rho\tilde{\rho}\,|n\rangle = \lambda_n^2\,|n\rangle \qquad \tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

N.B. "**convex roof function**": beware of *max* !

density matrix for pair of spins can be reconstructed from **equal-time correlations**

also works for pair of spins embedded in lattice

SU(2) symmetry:

$$C_{ij} = 2\max\left\{0, 2\,|\,g_{ij}^{zz}\,| - \left|\frac{1}{4} + g_{ij}^{zz}\right|\right\} \qquad g_{ij}^{zz} = \langle S_i^z S_j^z \rangle$$

C. H. Bennet *et al.*, Phys. Rev. A. **54**, 3824 (1996)
S. A. Hill *et al*, Phys. Rev. Lett. 7**8**, 5022 (1997)
W. K. Wooters, Phys. Rev. Lett. **80**, 2245 (1998)
R. Horodecki *et al*., Rev. Mod. Phys. **81**, 865 (2009)

what is quantum Fisher information ?

Quantum Fisher information (QFI) is a generalization of classical Fisher information (cf. Cramér-Rao bound) to the space of density matrices

QFI can be used to detect **multipartite entanglement** (k-producibility)

QFI vanishes if $[\rho, \hat{O}] = 0$

$$\rho = \sum_\lambda p_\lambda |\lambda\rangle\langle\lambda|$$

$$f_Q(\rho, \hat{O}) = \frac{2}{N} \sum_{\lambda,\lambda'} \frac{(p_\lambda - p_{\lambda'})^2}{p_\lambda + p_{\lambda'}} |\langle\lambda|\hat{O}|\lambda'\rangle|^2$$

$$f_Q(\rho, \hat{O}) > k \in \mathbb{Z}$$
$$\Rightarrow \text{depth of entanglement} \geq k + 1$$

$p_\lambda = \exp(-\beta E_\lambda)/Z$

operator of interest (*)

(*) spectrum of $\hat{O}$ assumed to be bounded on unit interval

QFI can be measured using **inelastic neutron scattering**:

dynamical structure factor

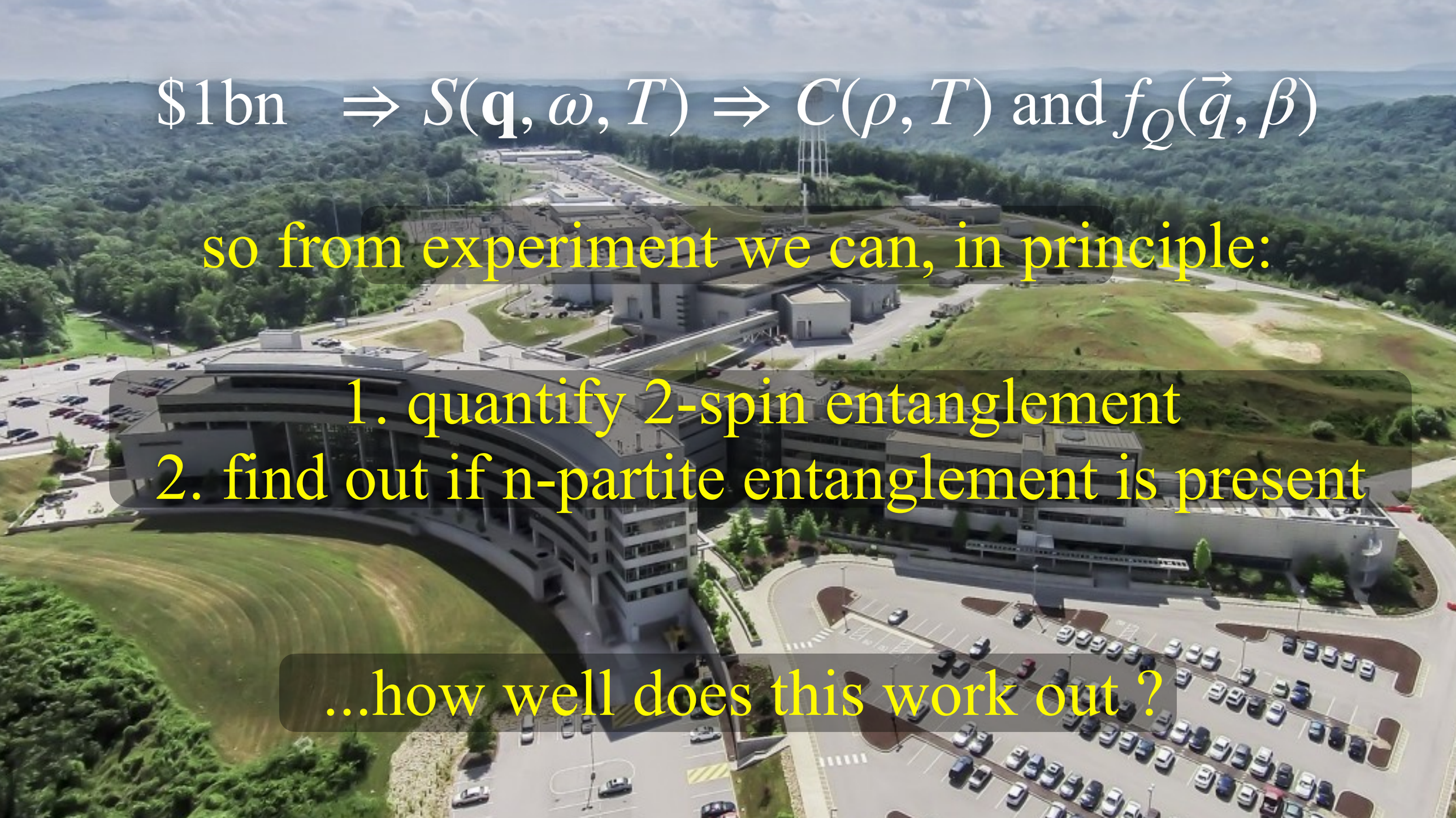
$$\hat{O}(\mathbf{q}) = \frac{1}{\sqrt{N}} \sum_i \boldsymbol{\sigma}_i \exp[-i\mathbf{r}_i \cdot \mathbf{q}]$$

$$f_Q(\vec{q}, \beta) = 4 \int_0^\infty d\omega \tanh\left(\frac{\beta\omega}{2}\right)(1 - e^{-\beta\omega}) S(\vec{q}, \omega, T)$$

QFI is a **robust experimental probe** of the depth of entanglement witnessed by spin fluctuations at a given **q**

most useful where $S(\mathbf{q}, \omega)$ has pronounced, temperature-dependent peak

S. L. Braunstein *et al.*, Phys. Rev. Lett. **72**, 3439 (1994)
P. Hyllus *et al.*, Phys. Rev. A **85**, 022321 (2012)
G. Toth, Phys. Rev. A **85**, 022322 (2012)
P. Hauke *et al.*, Nature Physics **12**, 778 (2015)


$1bn ⇒ $S(\mathbf{q}, \omega, T) \Rightarrow C(\rho, T)$ and $f_Q(\vec{q}, \beta)$
so from experiment we can, in principle:
1. quantify 2-spin entanglement
2. find out if n-partite entanglement is present
...how well does this work out ?



PHYSICAL REVIEW B **103**, 224434 (2021)

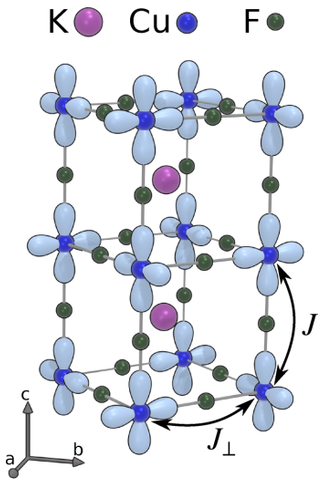
Editors' Suggestion

Witnessing entanglement in quantum magnets using neutron scattering

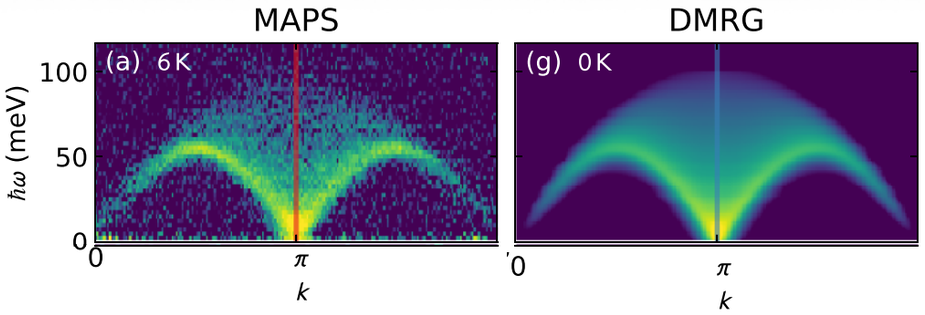
A. Scheie,[1,*,†] Pontus Laurell,[2,3,*,‡] A. M. Samarakoon,[1] B. Lake,[4,5] S. E. Nagler,[1,6] G. E. Granroth,[1] S. Okamoto,[7,6] G. Alvarez,[2,3] and D. A. Tennant[1,6,8,§]



$KCuF_3$



s=1/2 Heisenberg spin chain



calculate concurrence from equal time correlations:

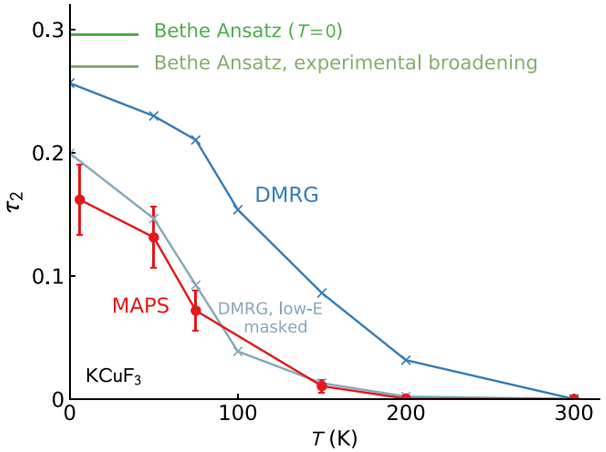
$$C_{ij} = 2\max\left\{0, 2\,|\,g^{zz}_{ij}\,| - \left|\frac{1}{4} + g^{zz}_{ij}\right|\right\} \qquad g^{zz}_{ij} = \langle S^z_i S^z_j \rangle$$

($SU(2)$ symmetry assumed)

evaluate "two-tangle":

$$\tau_2 = \sum_{j\neq i} C^2_{ij}$$

two-tangle as function of temperature



experimental results do not match simulation

N.B. model **extremely** well characterized: B. Lake *et al.*, Phys. Rev. Lett. **111**, 137205 (2013)



PHYSICAL REVIEW B **103**, 224434 (2021)

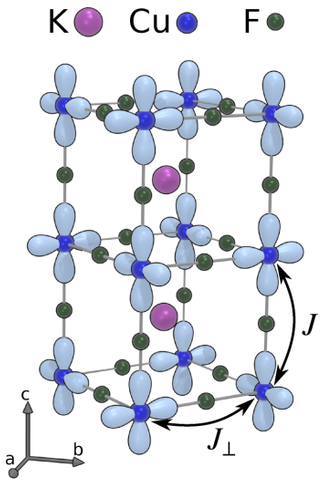
Editors' Suggestion

Witnessing entanglement in quantum magnets using neutron scattering

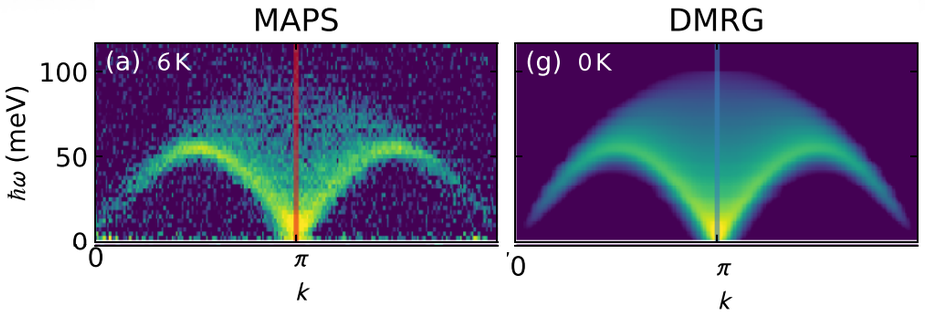
A. Scheie,[1,*,†] Pontus Laurell,[2,3,*,‡] A. M. Samarakoon,[1] B. Lake,[4,5] S. E. Nagler,[1,6] G. E. Granroth,[1] S. Okamoto,[7,6] G. Alvarez,[2,3] and D. A. Tennant[1,6,8,§]



$KCuF_3$



s=1/2
Heisenberg
spin chain



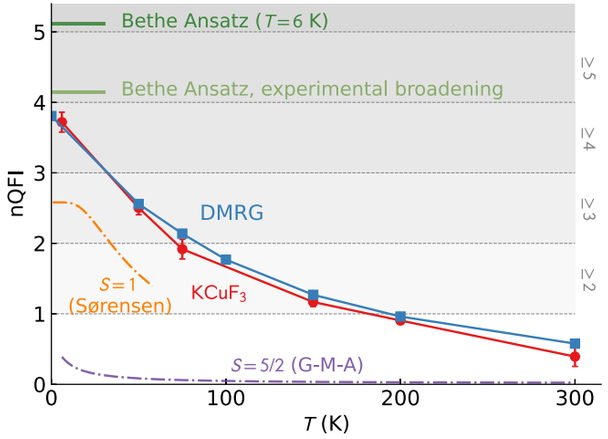
calculate QFI from dynamical structure factor

$$f_Q(T) = \frac{4\hbar}{\pi}\int_0^\infty d(\hbar\omega)\tanh\left(\frac{\hbar\omega}{2k_BT}\right)\chi''(\hbar\omega, T)$$

$$\chi''(\omega) = \pi(1 - e^{-\hbar\omega\beta})S(\omega),$$

A. Scheie *et al*, Phys. Rev. B **107**, 059902(E) (2023)
Erratum: A. Scheie *et. al.*, Phys. Rev. B **107**, 059902 (2023)

QFI as function of temperature



evidence for multipartite entanglement, growing at low-T



nature physics

Article

https://doi.org/10.1038/s41567-023-02259-1

Proximate spin liquid and fractionalization in the triangular antiferromagnet $KYbSe_2$

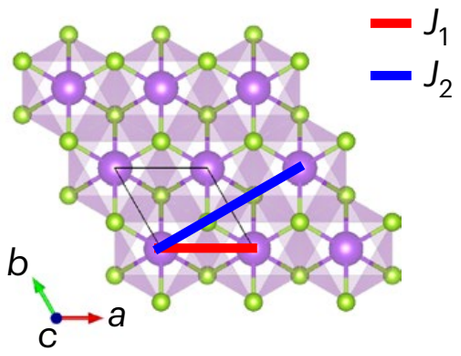
Received: 23 September 2021

Accepted: 14 September 2023

Published online: 6 November 2023

Check for updates

A. O. Scheie[1], E. A. Ghioldi[2,3], J. Xing[4], J. A. M. Paddison[4], N. E. Sherman[5,6], M. Dupont[5,6], L. D. Sanjeewa[7,8], Sangyun Lee[9], A. J. Woods[9], D. Abernathy[1], D. M. Pajerowski[1], T. J. Williams[1], Shang-Shun Zhang[10], L. O. Manuel[3], A. E. Trumper[3], C. D. Pemmaraju[11], A. S. Sefat[4], D. S. Parker[4], T. P. Devereaux[11,12], R. Movshovich[9], J. E. Moore[5,6], C. D. Batista[2,13] & D. A. Tennant[2,13]



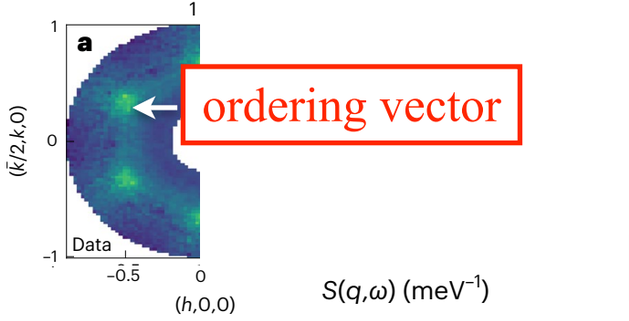
$KYbSe_2$

Yb^{3+} has pseudo-spin 1/2 ground-state doublet

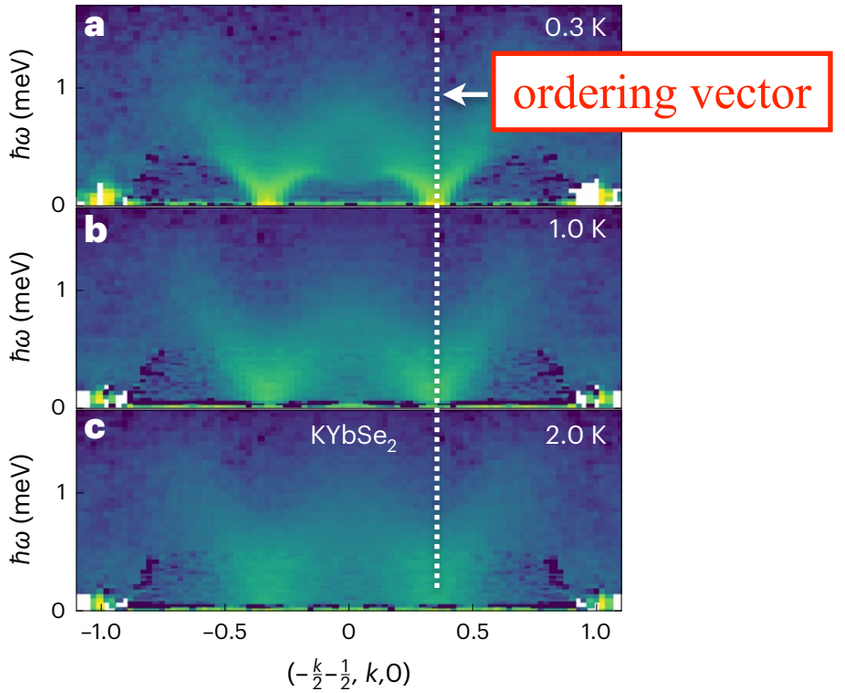
triangular-lattice AF with 120° Néel order (ordered moment $m_s \approx 0.58\mu_B$)

minimal model

$$\mathscr{H} = J_1 \sum_{\langle ij \rangle_1} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle ij \rangle_2} \mathbf{S}_i \cdot \mathbf{S}_j$$



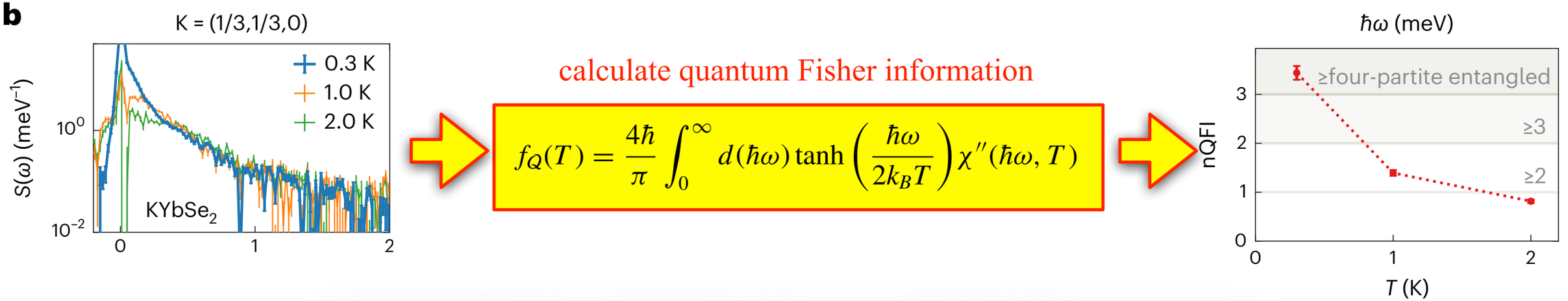




neutron scattering shows diffuse scattering with broad continuum of excitations

A. Scheie *et al.*, Nat. Phys. **20**, 74 (2024)

what depth of entanglement does this imply ?



spin liquid[55]). Meanwhile, the QFI shows at least bipartite entanglement within the (1/3, 1/3) correlations. Both of these rule out classical glassiness or random singlet formation. Instead, they point to many sites entangled together at the lowest temperatures—as one would expect for a QSL.

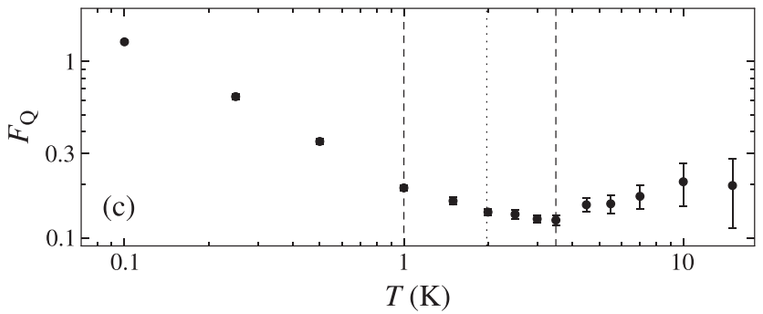
nQFI > 2 assumed to rule out disorder-driven random singlet state

A. Scheie *et al.*, Nat. Phys. **20**, 74 (2024)

what is seen elsewhere ?

$YbZnGaO_4$

triangular lattice AF
and QSL candidate



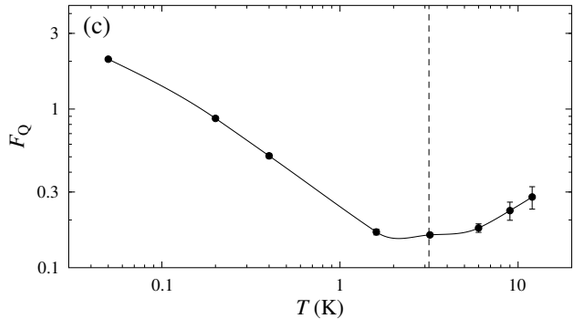
QFI measured in muSR shows
upturn at low-T

F. Pratt *et al.*,
Phys. Rev. B. **106**, L060401 (2022)

+

$YbZnGaO_4$

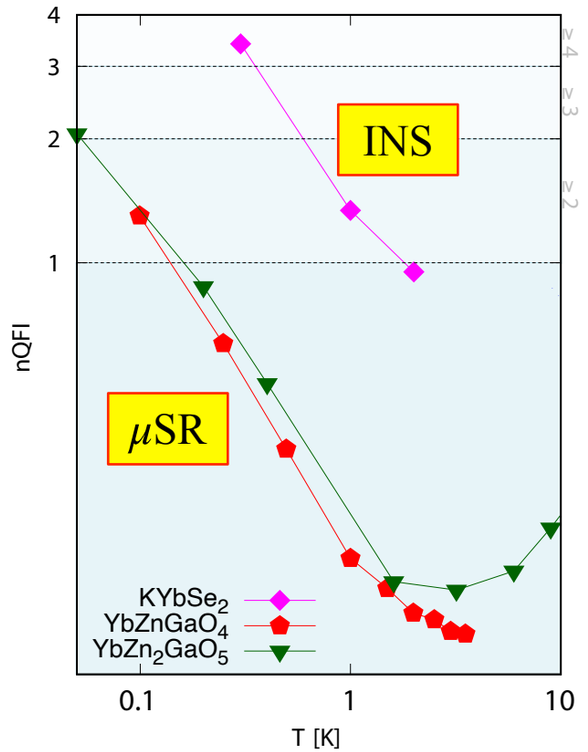
new triangular lattice AF
and QSL candidate



QFI measured in muSR shows
upturn at low-T

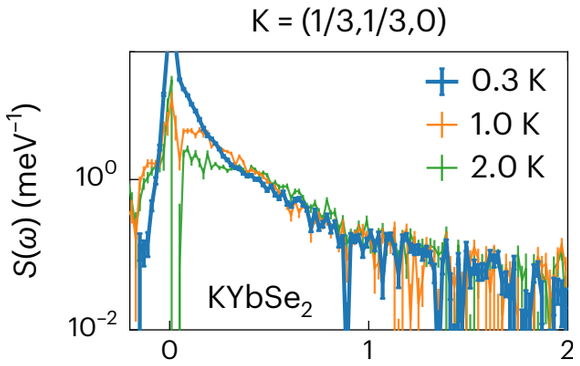
H.C.H. Wu *et al.*,
arXiv:2502.00130

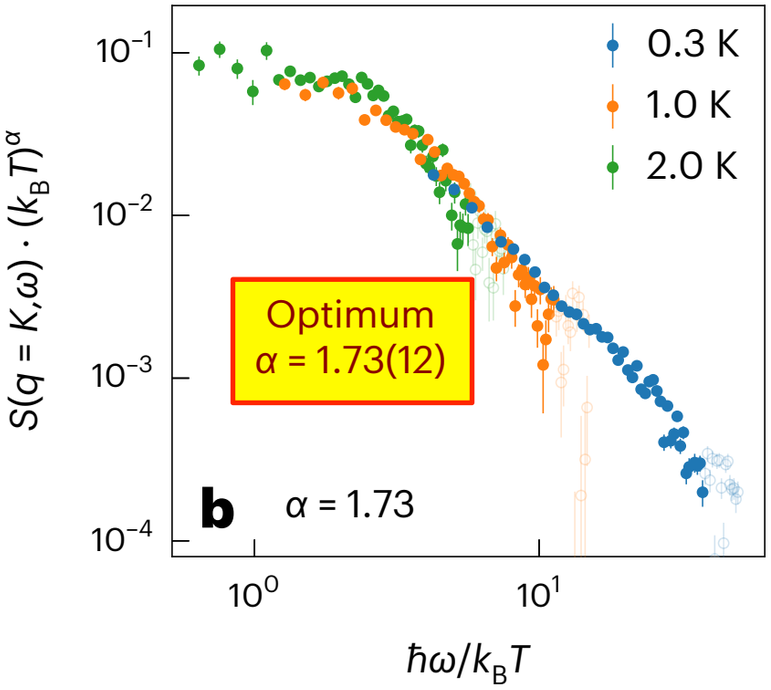
all 3 materials show power-law
scaling of QFI at low-T

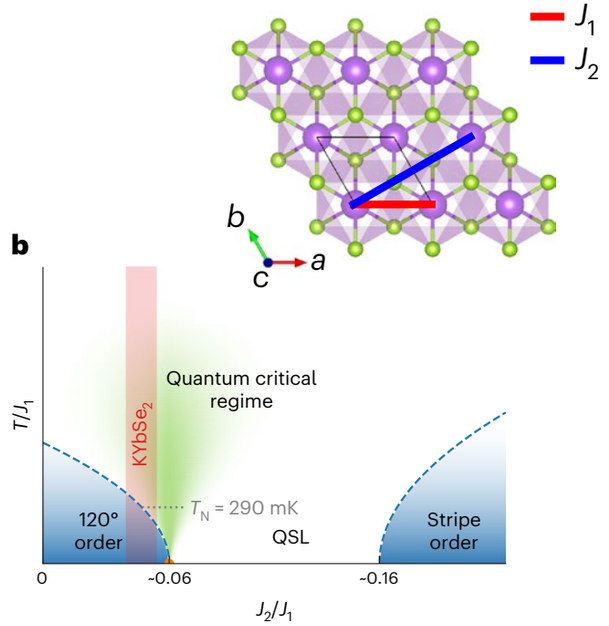


T. Shimokawa *et al.*,
arXiv:2505.11874

what else do fluctuations in $KYbSe_2$ reveal ?







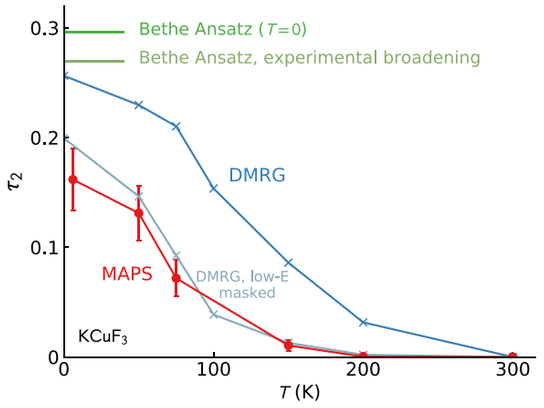
continuum
of excitations
at ordering vector

scaling collapse as function $\hbar\omega/k_BT$
⇒ nearby quantum critical point (QCP) ?

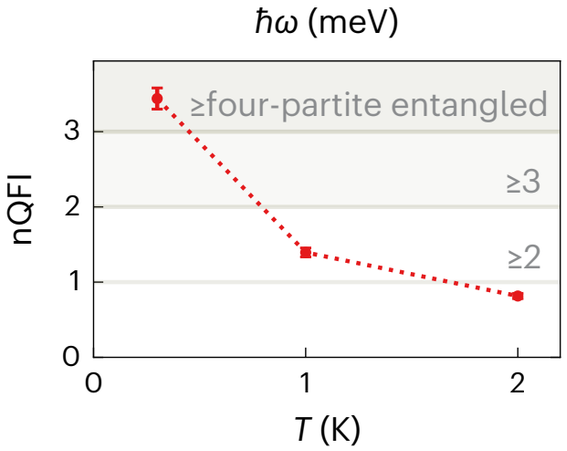
interpreted in terms of J_1-J_2
Heisenberg model with
$J_2/J_1 \approx 0.047$

A. Scheie *et al.*, Nat. Phys. **20**, 74 (2024)

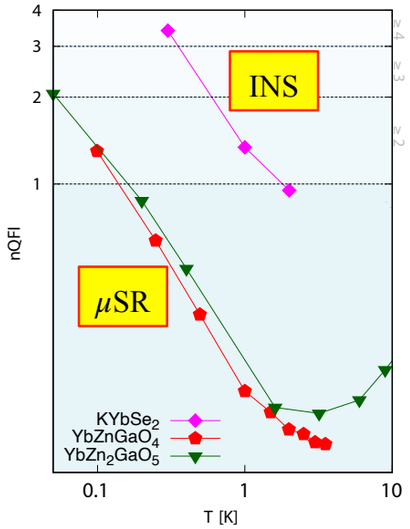
what do we need to understand ?



why do experimental results for 2-tangle in $KCuF_3$ differ from simulations ?

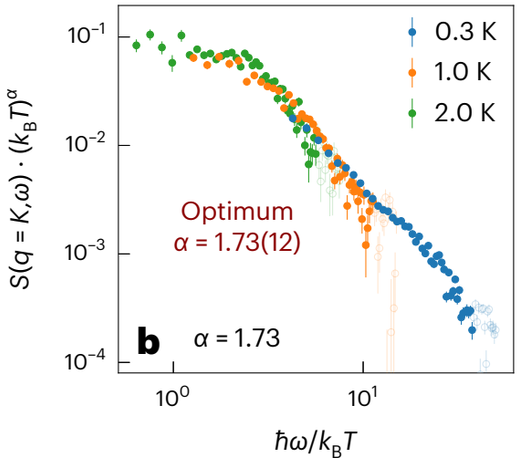


is it safe to rule out random singlet phases on the grounds of multipartite entanglement ?



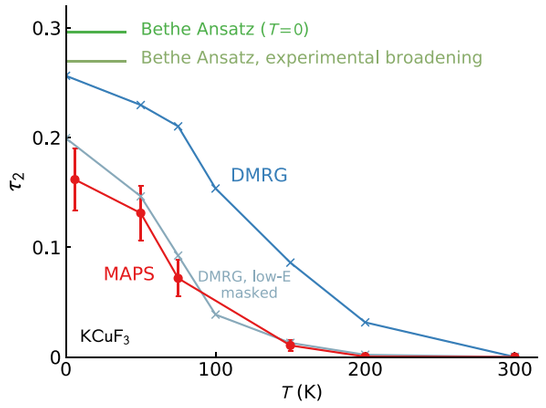
how should we interpret the power-law scaling of QFI seen in triangular lattice AF's ?

bonus question

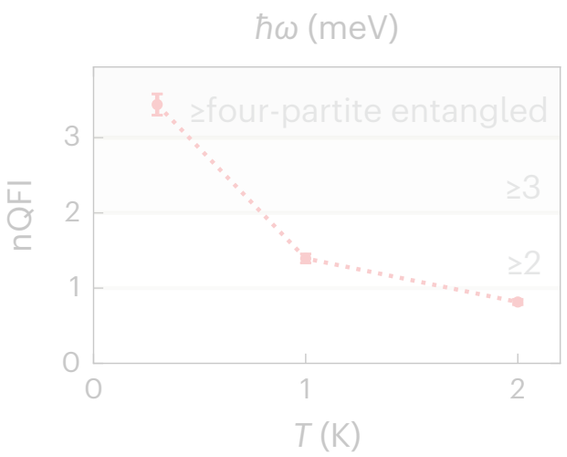


can we understand the way in which fluctuations scale with energy and temperature in the triangular lattice AF $KYbSe_2$?

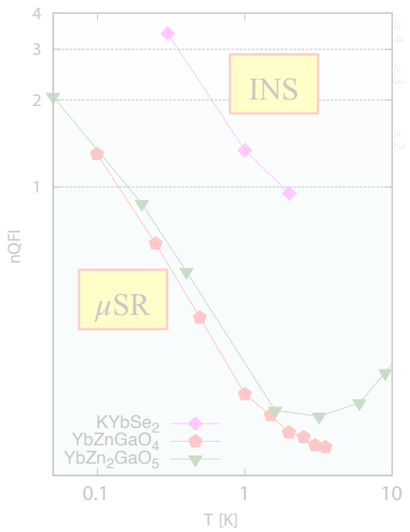
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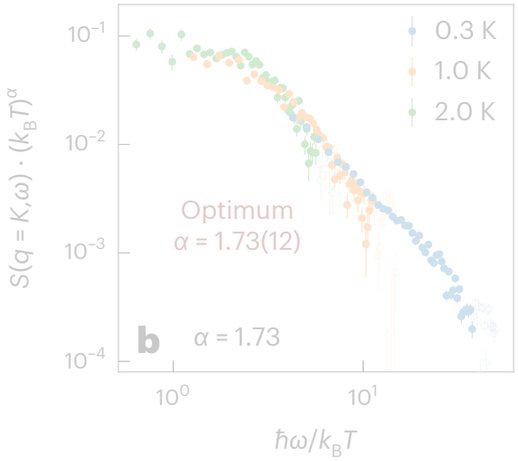


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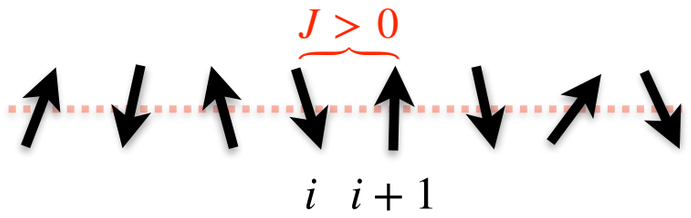
bonus question



can we understand the way in which fluctuations scale with energy and temperature in the triangular lattice AF $KYbSe_2$?

what happens to concurrence in presence of disorder ?

consider spin-1/2 AF Heisenberg spin chain with quenched random bond-disorder



$$\mathscr{H} = \sum_i J_i \, \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

$$J_i \in [J - \Delta, J + \Delta]$$

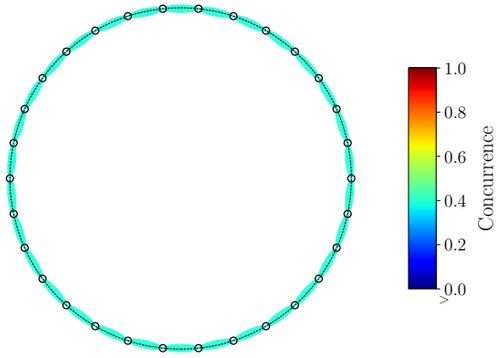


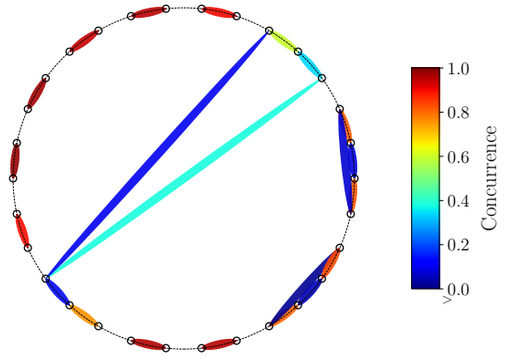
calculate concurrence for finite spin chain using ED

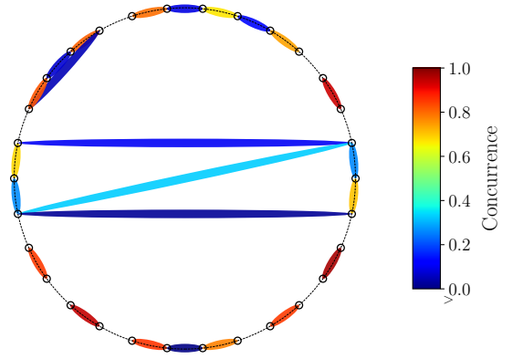
$$C_{ij} = 2 \max \left\{ 0, 2\,|\, g_{ij}^{zz} \,| - \left| \frac{1}{4} + g_{ij}^{zz} \right| \right\} \qquad g_{ij}^{zz} = \langle S_i^z S_j^z \rangle$$

in TLL ($\Delta = 0$), concurrence vanishes on all but 1st neighbour bonds

in RS phase ($\Delta \neq 0$), concurrence take on a finite value on longer range bonds

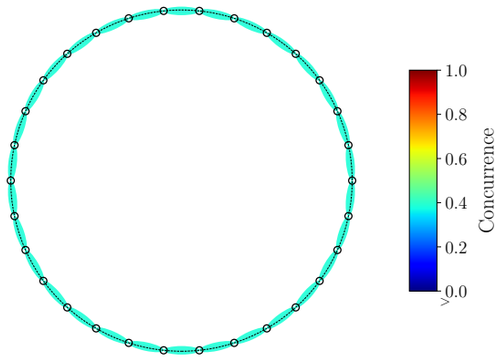




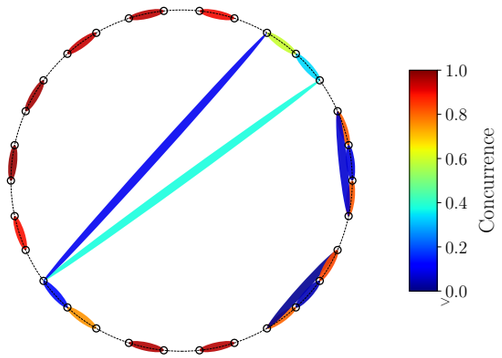


S. Sabharwal *et al.*, Phys. Rev. Res. **7**, 023271 (2025)

can we use this to distinguish TLL and RSS ?



v.



...there is a problem



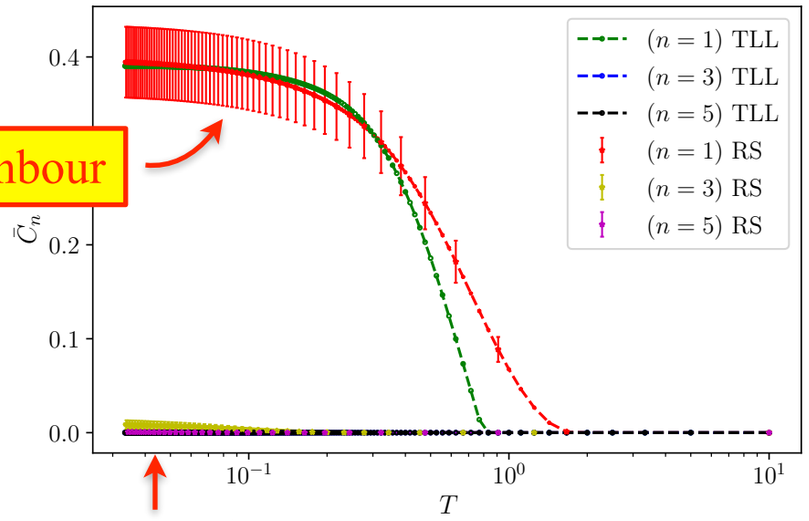
order of averages matters !

concurrence calculated as ensemble average over correlations within different realizations of disorder
(cf. theory)

$$\bar{C}_{ij} = 2\max\left\{0, 2\,|\,g_{ij}^{zz}\,| - \left|\frac{1}{4} + g_{ij}^{zz}\right|\right\}$$

concurrence calculated as average of correlations in the presence of disorder
(cf. experiment)

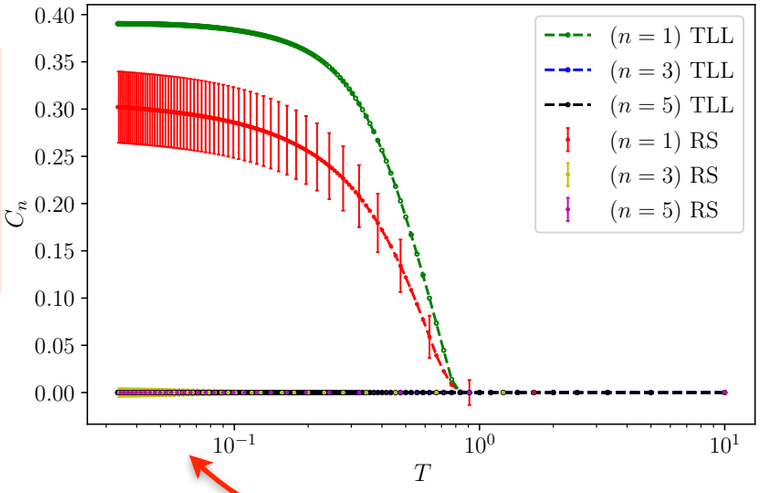
$$C_{ij} = 2\max\left\{0, 2\,|\,\bar{g}_{ij}^{zz}\,| - \left|\frac{1}{4} + \bar{g}_{ij}^{zz}\right|\right\}$$



1st neighbour

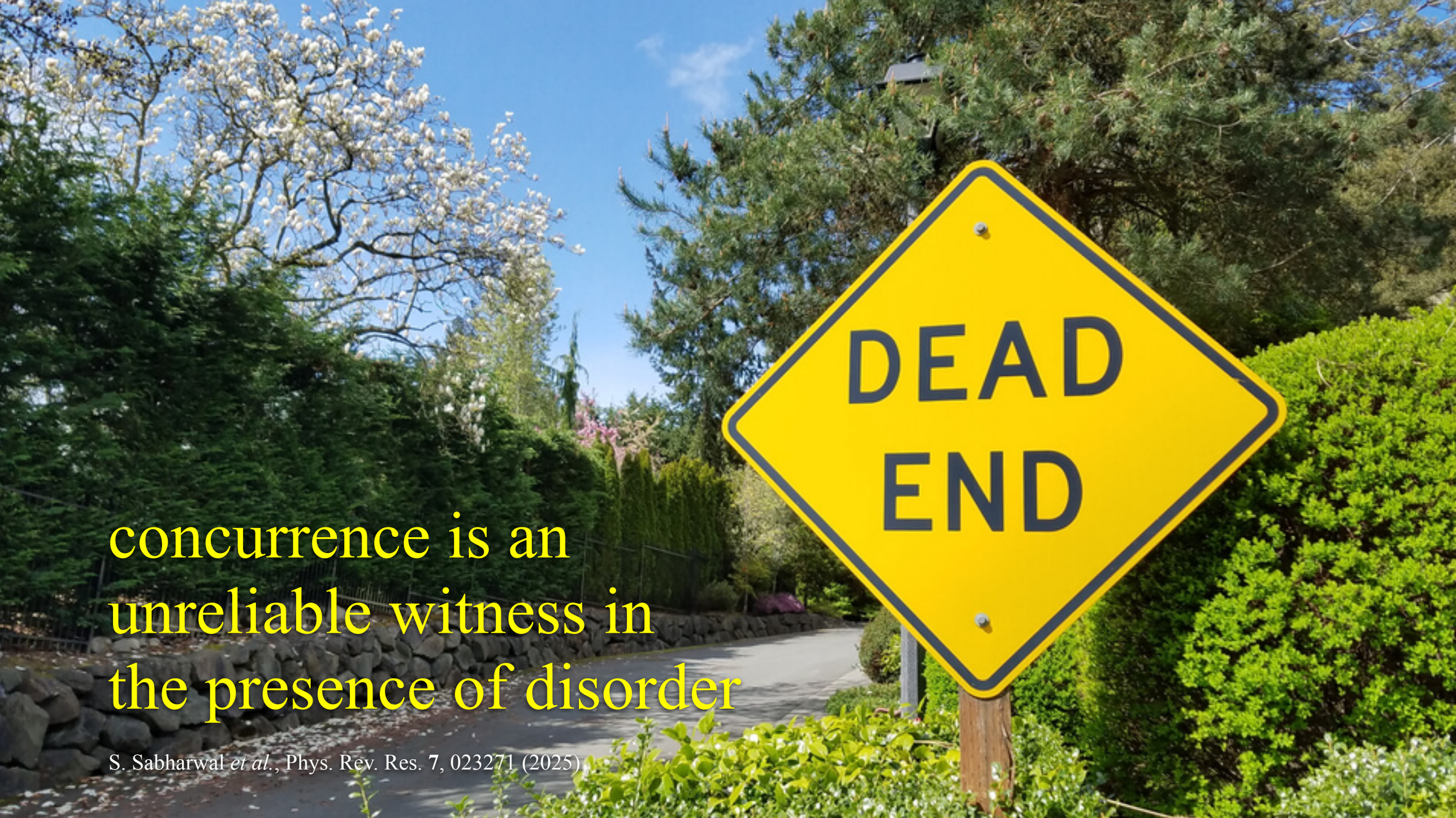
3rd-neighbour, 5th neighbour

systematic differences due to convex roof function



qualitative distinction between TLL and RSS is lost...

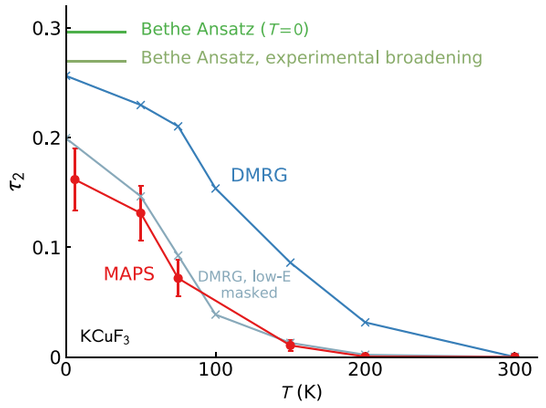
S. Sabharwal *et al.*, Phys. Rev. Res. **7**, 023271 (2025)



concurrence is an unreliable witness in the presence of disorder

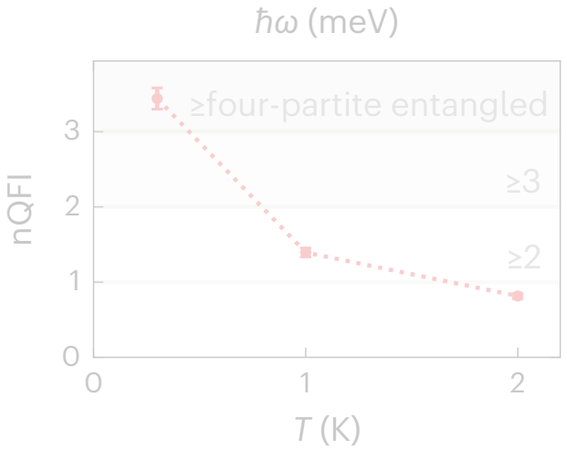
S. Sabharwal *et al.*, Phys. Rev. Res. **7**, 023271 (2025)

what do we need to understand ?

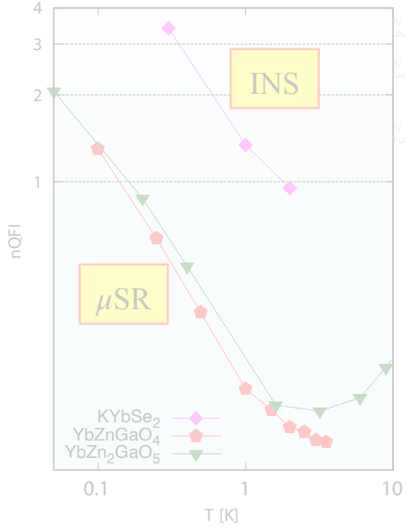


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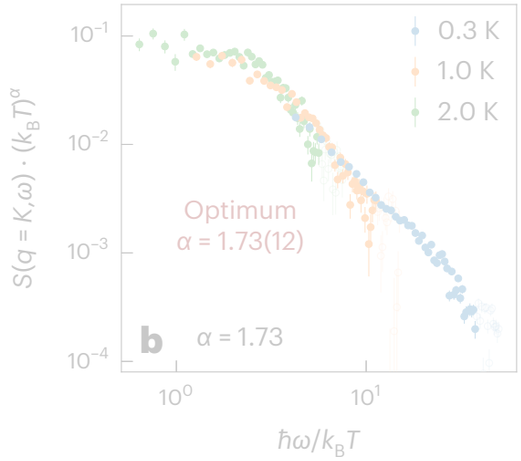


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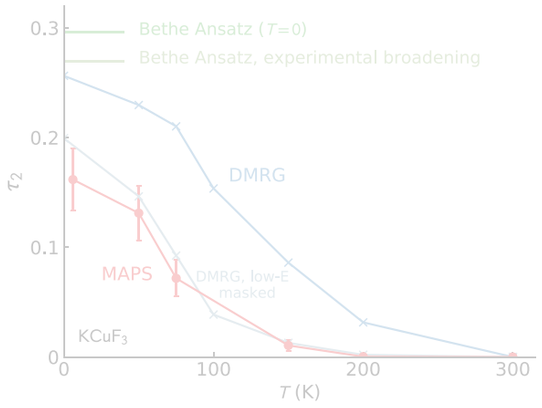
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bonus question



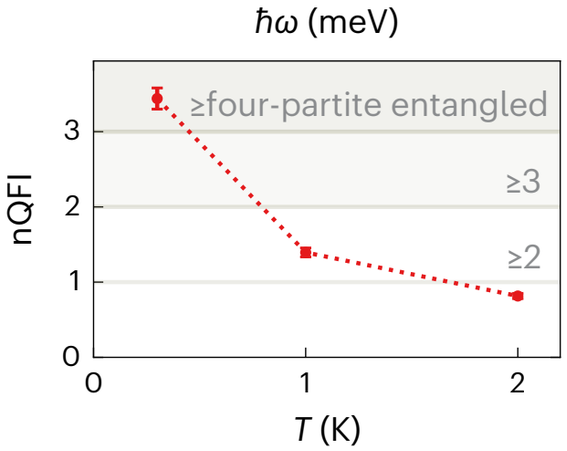
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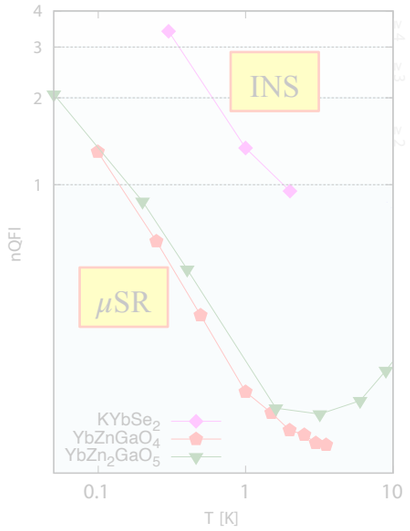


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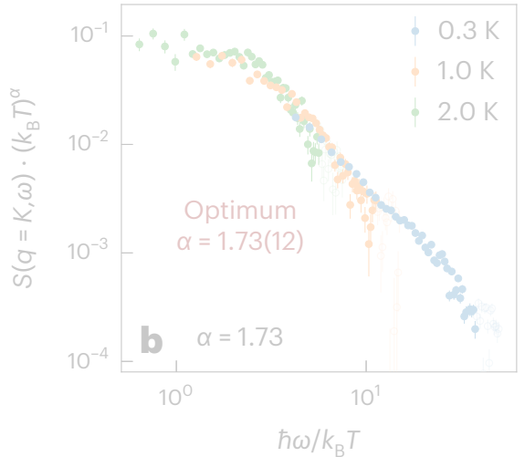


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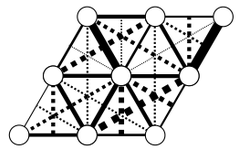


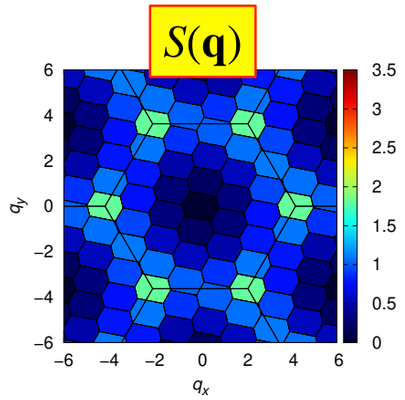
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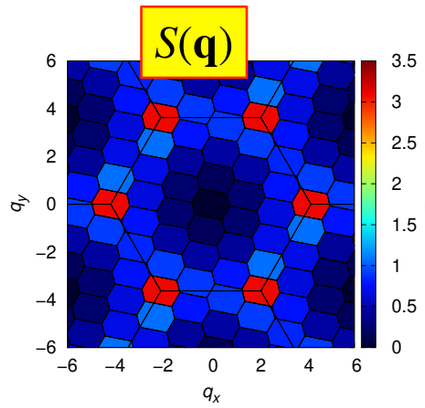
can we distinguish phases by correlations alone ?

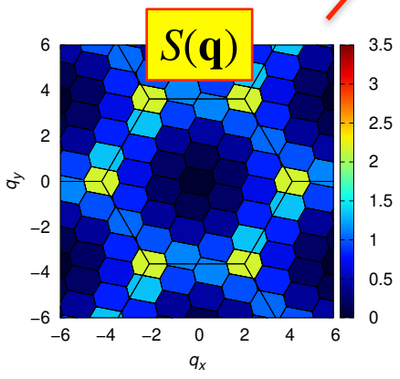
spin-1/2 J_1-J_2 Heisenberg model with quenched random bond disorder

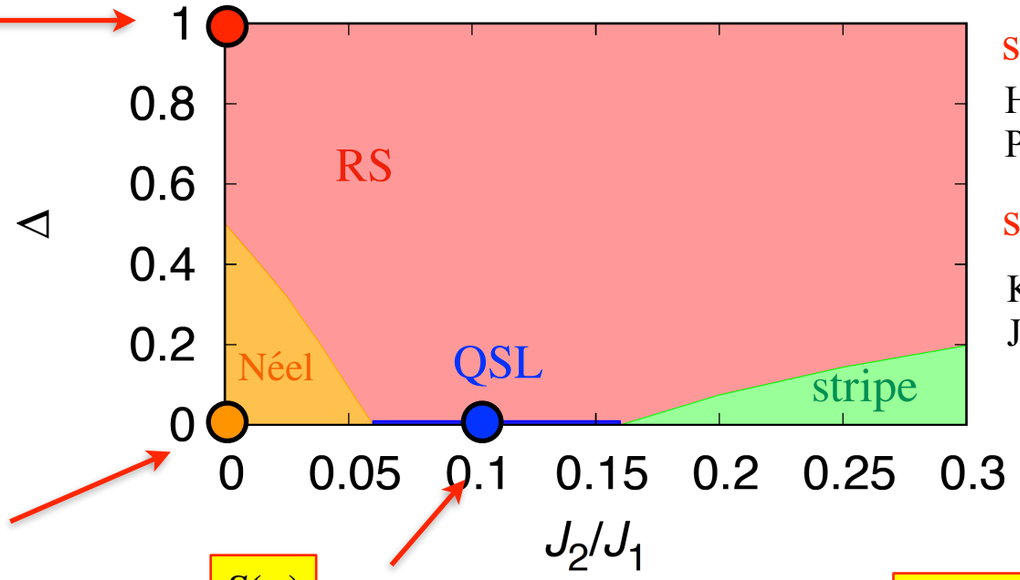
$$\mathscr{H} = J_1 \sum_{\langle ij \rangle_1} (1 + \Delta\alpha_{ij}) \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle ij \rangle_2} (1 + \Delta\beta_{ij}) \mathbf{S}_i \cdot \mathbf{S}_j \qquad [\alpha_{ij}, \beta_{ij} \in (-1,1)]$$











schematic phase diagram cf.

H.-Q. Wu *et. al*,
Phys. Rev. B **99**, 085141 (2019)

see also work of Kawamura group, e.g.

K. Watanabe *et al.*,
J. Phys. Soc. Jpn **83**, 034714 (2013)

Néel, Quantum Spin Liquid (QSL) and random singlet (RS) phases are poorly differentiated by S(**q**)

T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874

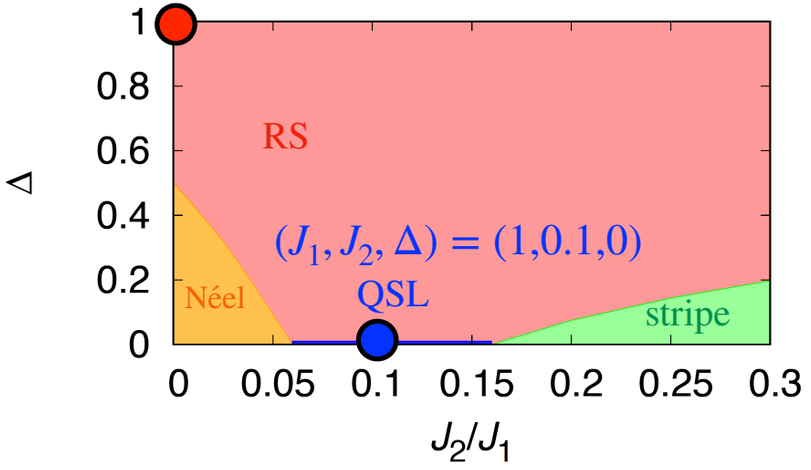


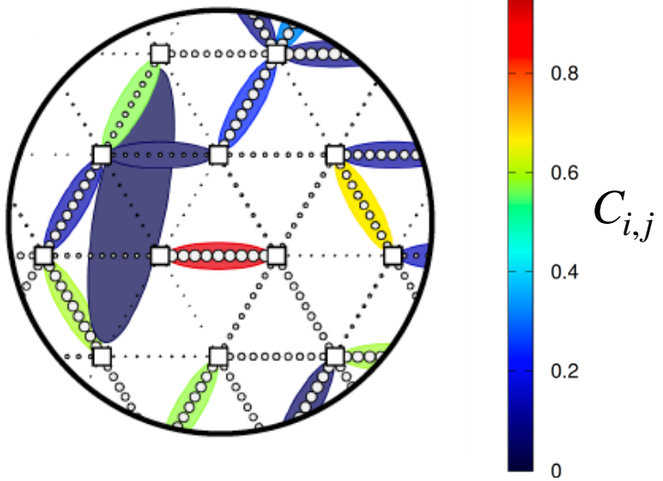
what does concurrence tell us ?

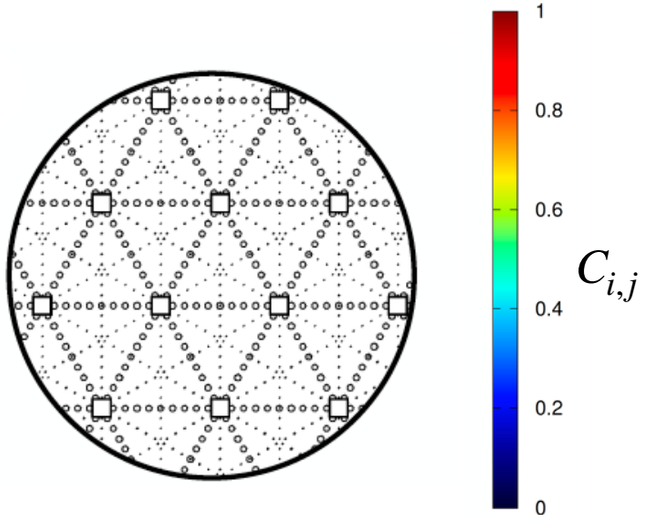
RS phase

QSL phase

$(J_1, J_2, \Delta) = (1,0,1)$







$$\tau_j^{(2)} = \sum_{i(\neq j)} C_{i,j}^2$$

$\tau^{(2)} > 0$
(pairs of spins entangled)

$\tau^{(2)} = 0$
zero two-spin entanglement

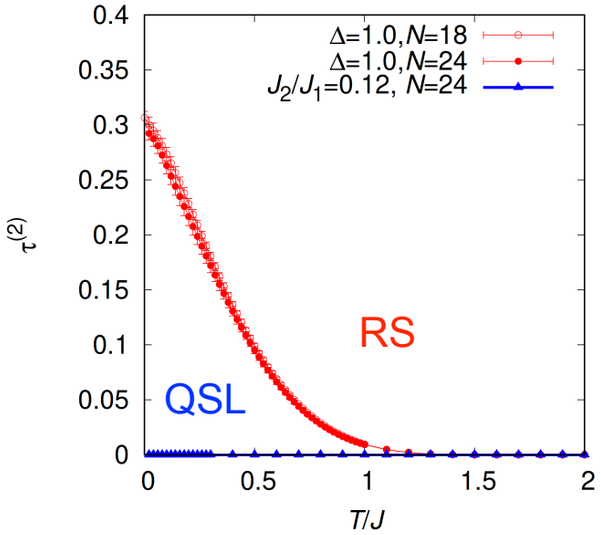
T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874



can we see this is experiment ?

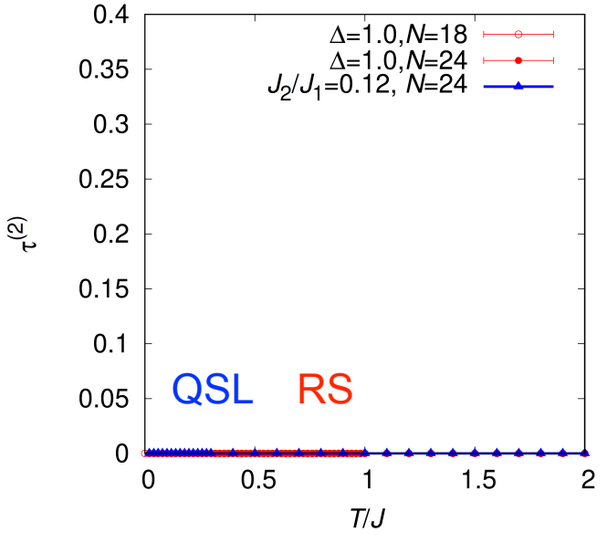
concurrence calculated as ensemble average over correlations within different realizations of disorder
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$$\bar{C}_{ij} = 2\max\left\{0, 2\,|\,g_{ij}^{zz}\,| - \left|\frac{1}{4} + g_{ij}^{zz}\right|\right\}$$



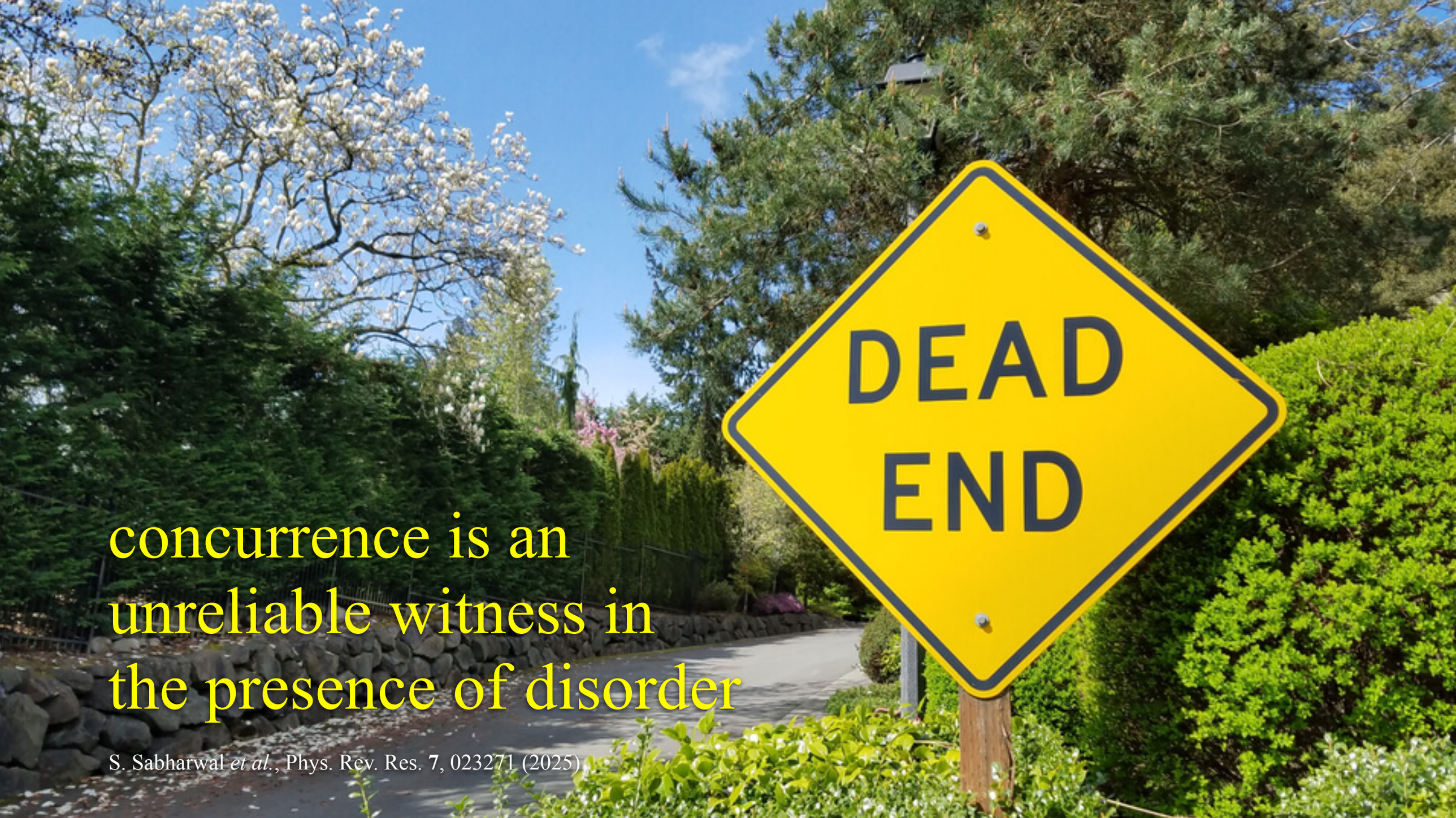
concurrence calculated as average of correlations in the presence of disorder
(cf. experiment)

$$C_{ij} = 2\max\left\{0, 2\,|\,\bar{g}_{ij}^{zz}\,| - \left|\frac{1}{4} + \bar{g}_{ij}^{zz}\right|\right\}$$



two-tangle, via INS, is completely useless for distinguishing between RS and QSL…

T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874

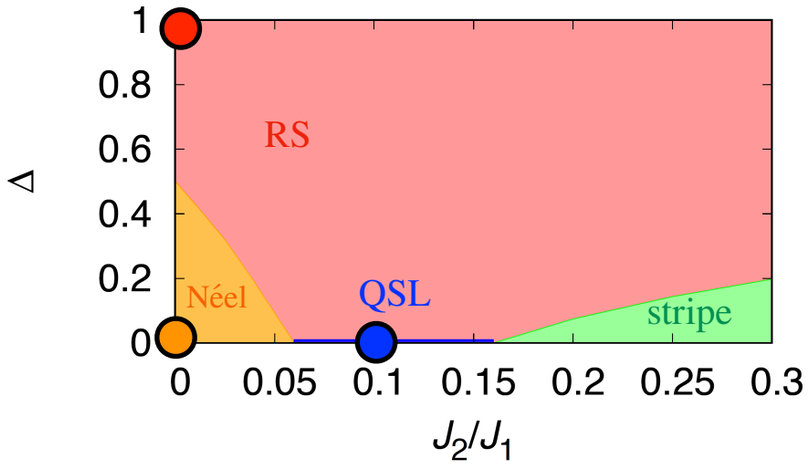


concurrence is an unreliable witness in the presence of disorder

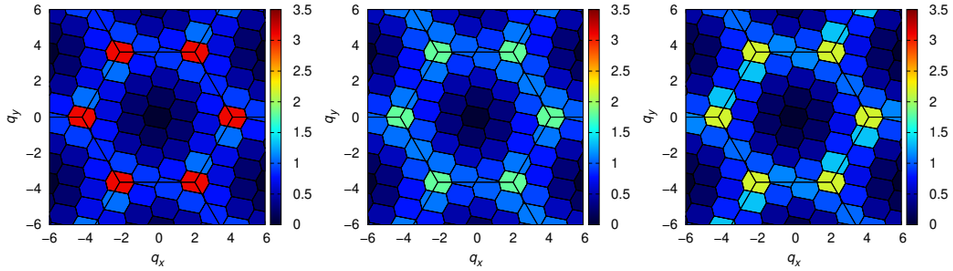
S. Sabharwal *et al.*, Phys. Rev. Res. **7**, 023271 (2025)



what about quantum Fisher information ?



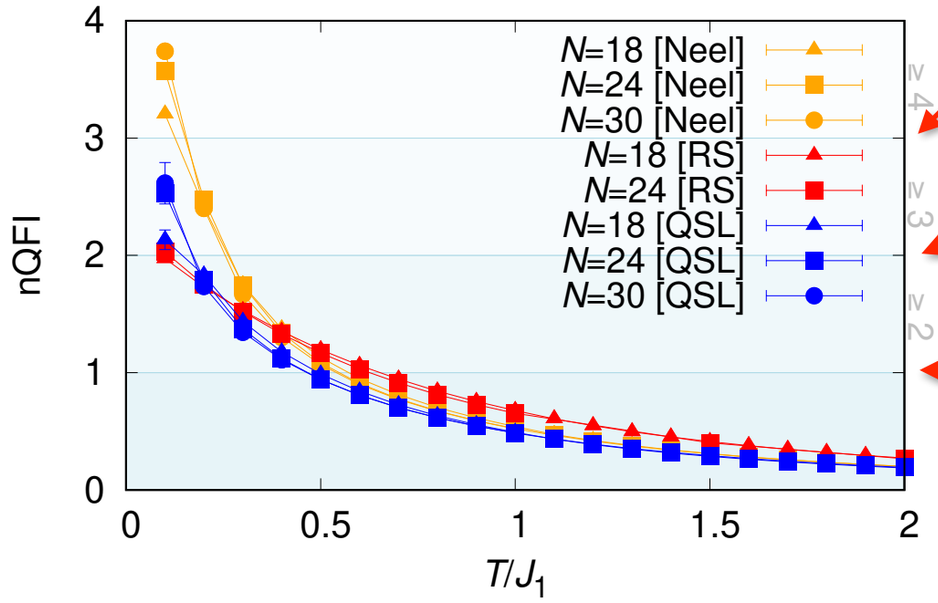
Néel QSL RS



strongest fluctuations at $\mathbf{q} = K$

⇒ evaluate QFI at this wavevector

$S(\mathbf{q}_K, \omega, T)$ found using thermal pure quantum states (TPQ) ⇒ calculate QFI



entangled quadrupules of spins

entangled triples of spins

entangled pairs of spins

QSL, Néel and RS phases all possess multiparty entanglement at low temperature

T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874

should we be surprised ?

consider Heisenberg model describing collinear Néel antiferromagnet

$$\mathscr{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle ij \rangle} \left[S_i^z S_j^z + \frac{1}{2} \underbrace{\left(S_i^+ S_j^- + S_i^- S_j^+ \right)}_{\textit{mixing term}} \right] \qquad J > 0$$

at T=0, recover ground state

$$\mathscr{H} \mid \Psi_0 \rangle = E_0 \mid \Psi_0 \rangle$$

$$\mid \Psi_0^{(\mathrm{Ising})} \rangle = \underbrace{\mid \uparrow \rangle_1 \otimes \mid \downarrow \rangle_2 \otimes \mid \uparrow \rangle_3 \otimes \ldots \mid \downarrow \rangle_N}_{\textit{not eigenstate of Heisenberg model}}$$

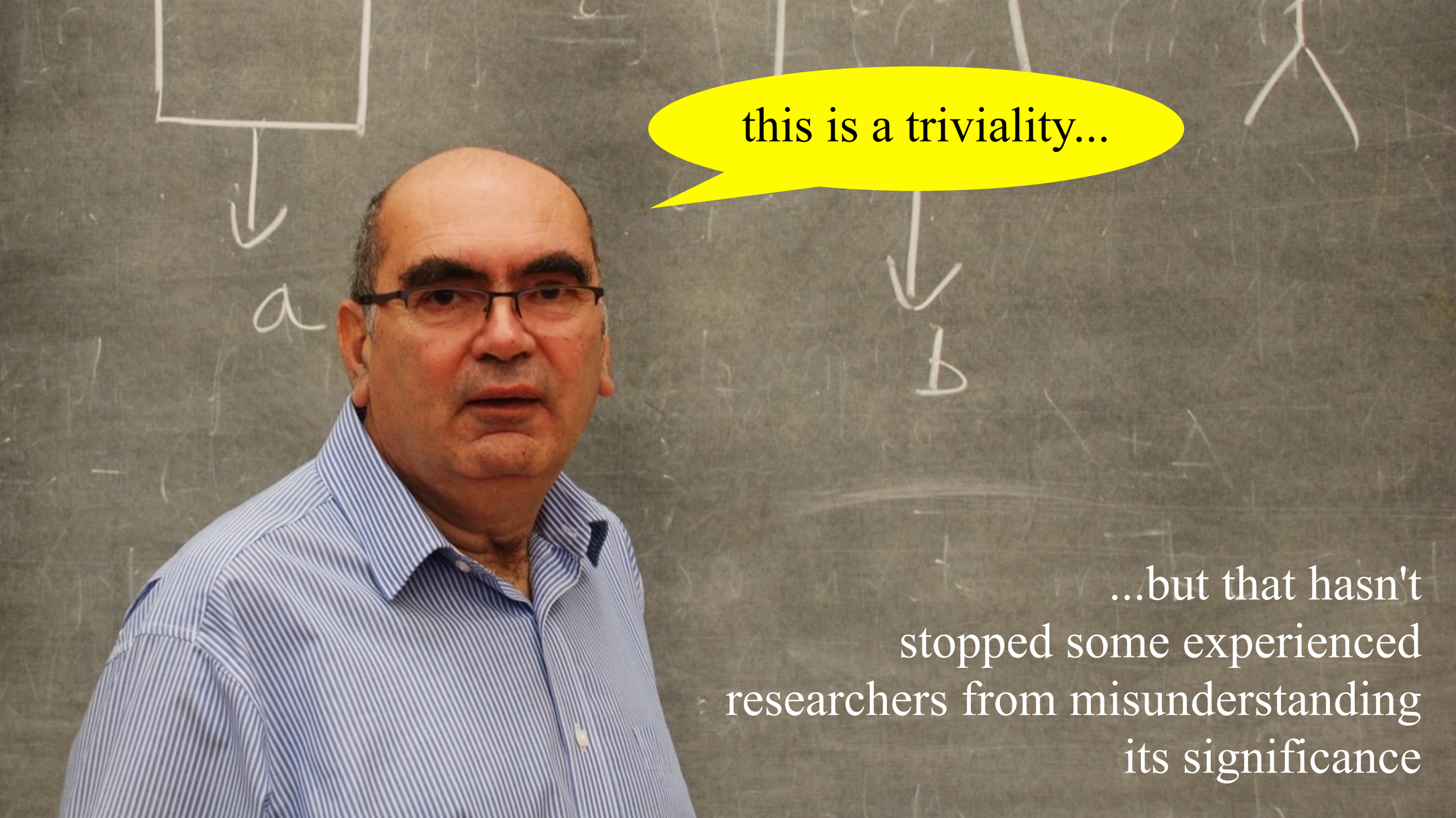
...ground state of Ising model is simple product state
(classical spin configuration with lowest energy)

product states are (almost) never groundstates of realistic Hamiltonians...

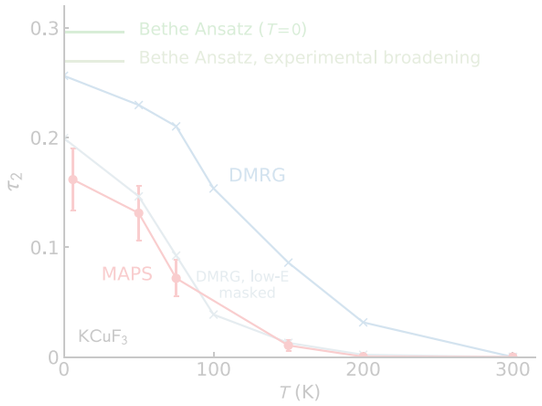


$$\mid \Psi_{AB} \rangle = \sum_{n,m} c_{nm} \mid n \rangle_A \otimes \mid m \rangle_B \neq \mid \Psi_A \rangle \otimes \mid \Psi_B \rangle \qquad c_{nm} \geq 0$$

...at low enough temperatures we should always expect to find a finite depth of entanglement

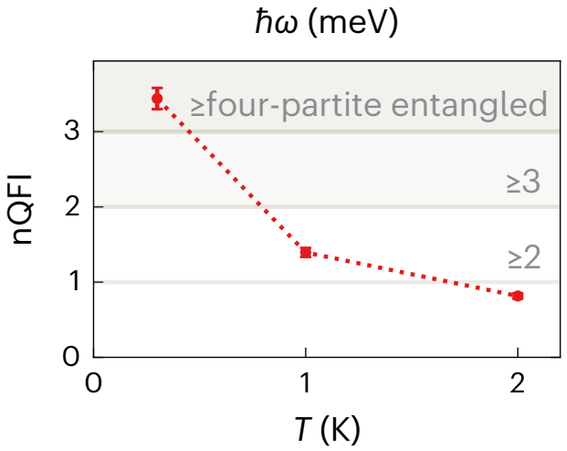

this is a triviality...
a
b
...but that hasn't
stopped some experienced
researchers from misunderstanding
its significance

what do we need to understand ?



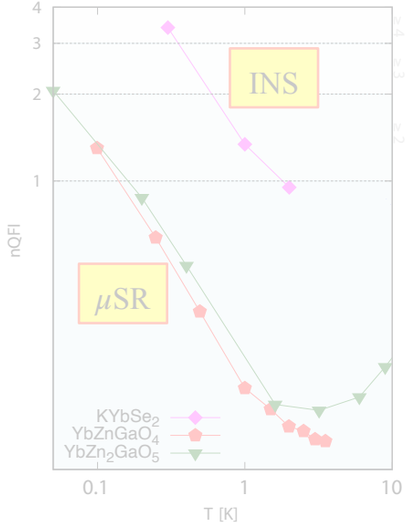
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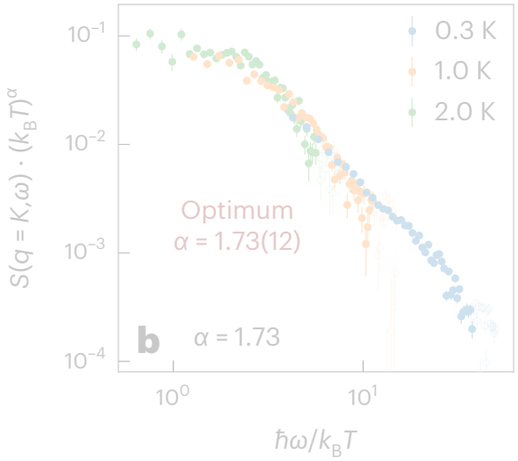
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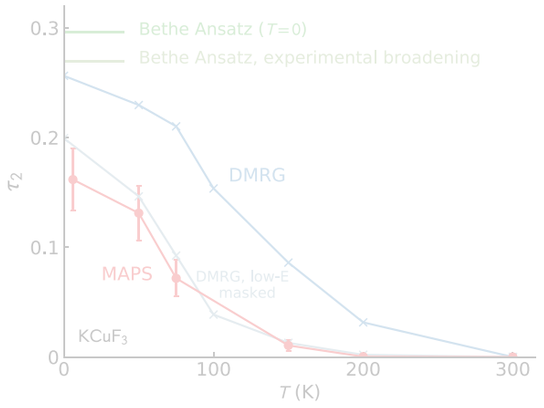
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bonus question



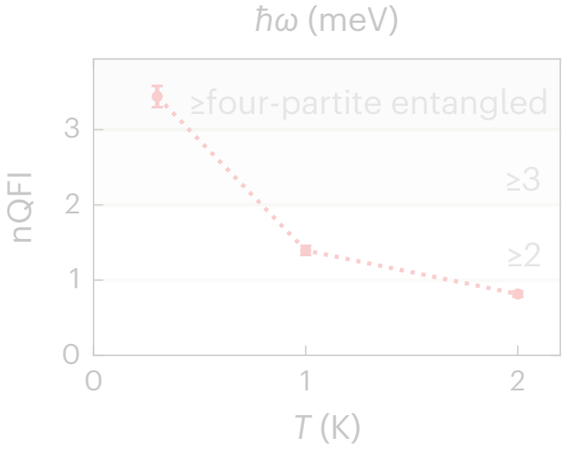
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what do we need to understand ?



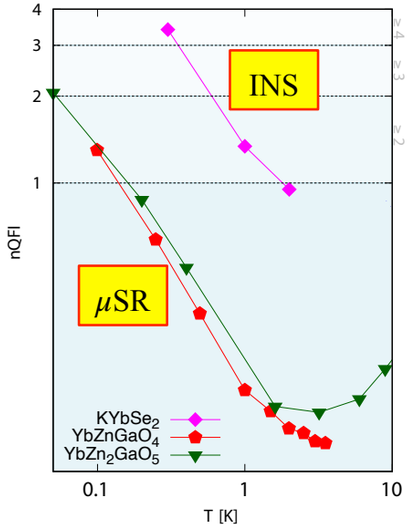
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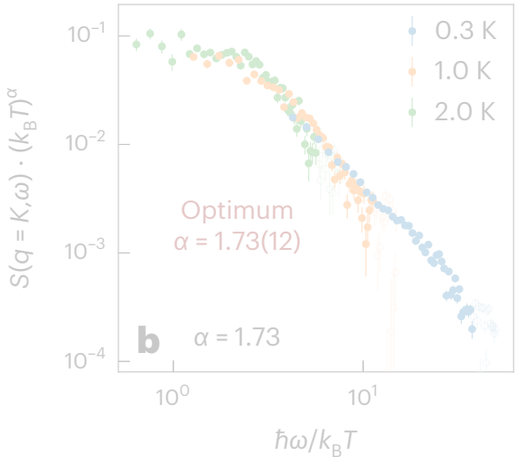
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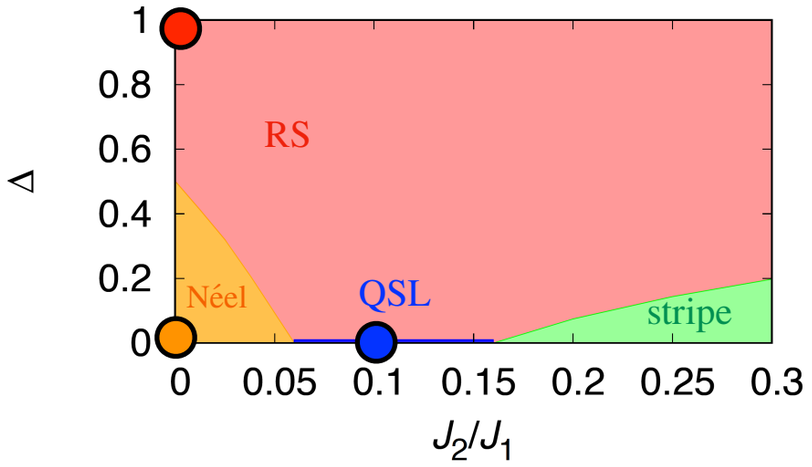
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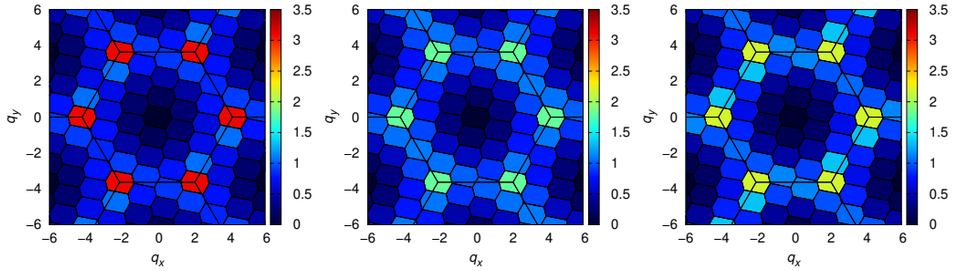
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how does QFI scale with temperature ?



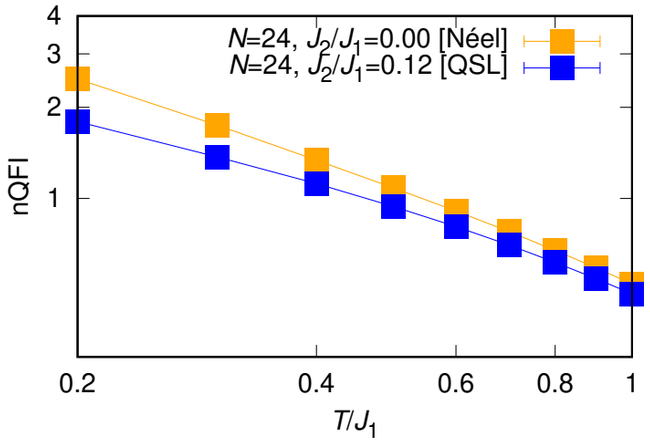
Néel QSL RS

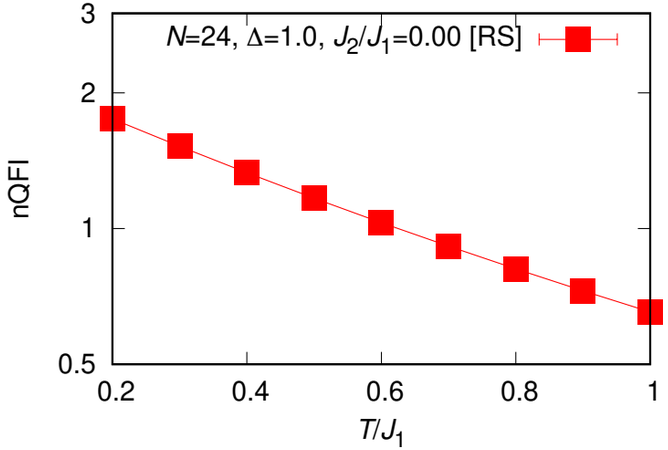


strongest fluctuations at $\mathbf{q} = K$

⇒ evaluate QFI at this wavevector

temperature-dependence of QFI distinguishes RS from QSL and Néel phases





$$f_{\mathcal{Q}}[\mathbf{q}_K, T/J_1] \propto b(T/J_1)^{-c}$$

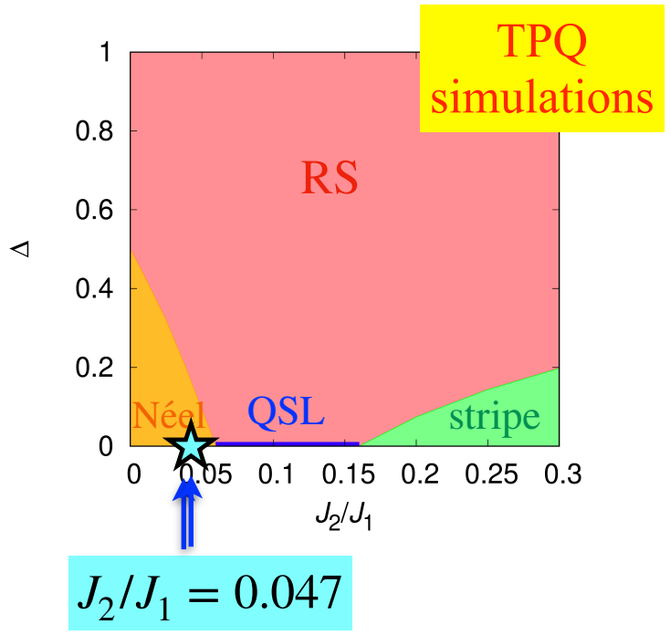
QSL and Néel show power-law scaling of QFI as function T/J

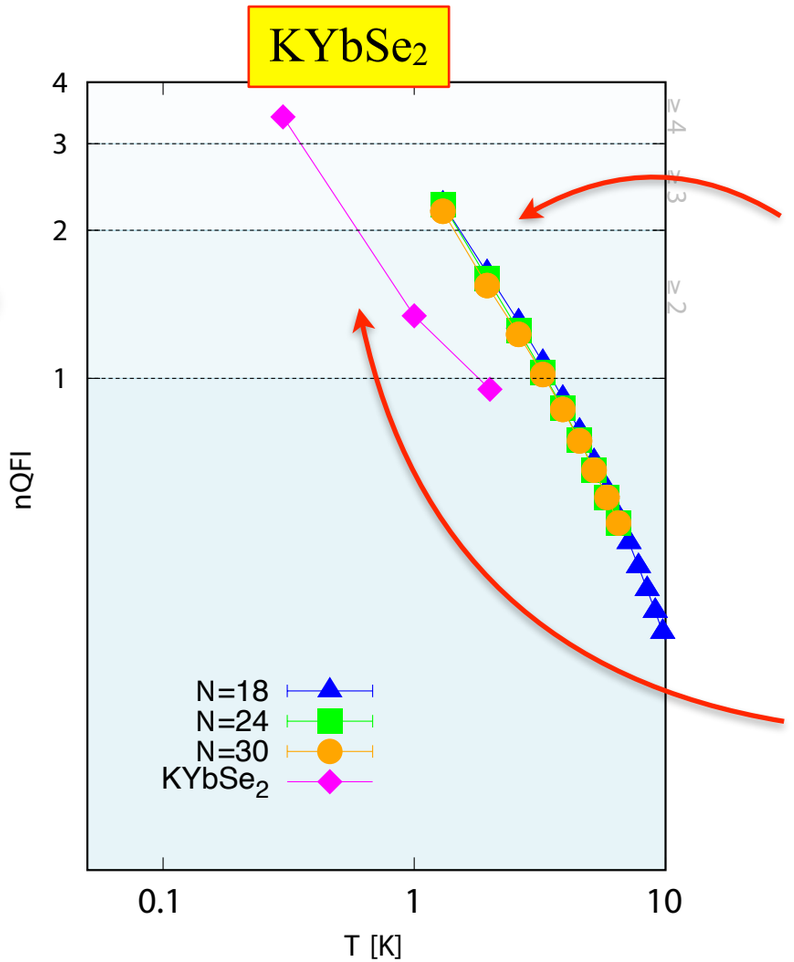
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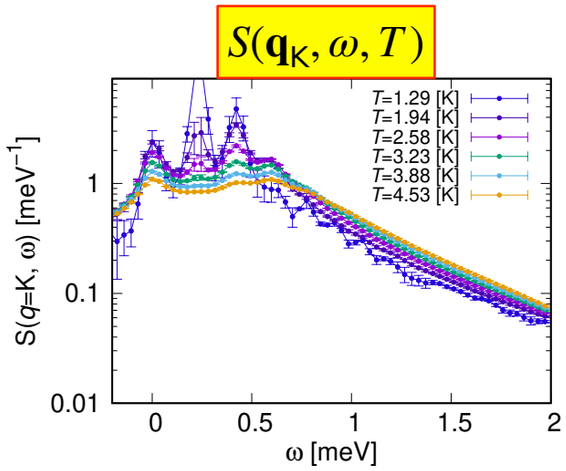
RS shows exponential decay as function T/J

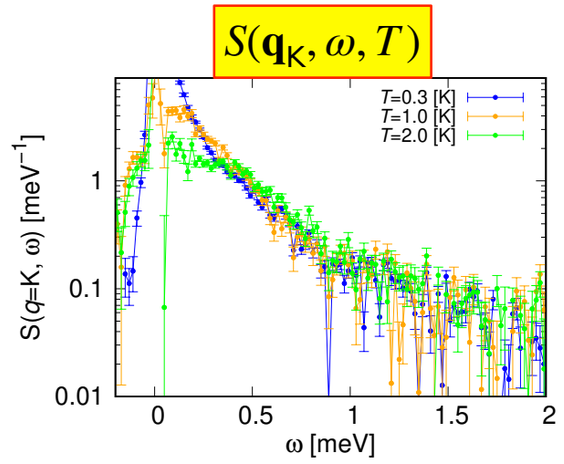
T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874

how does this compare with $KYbSe_2$?





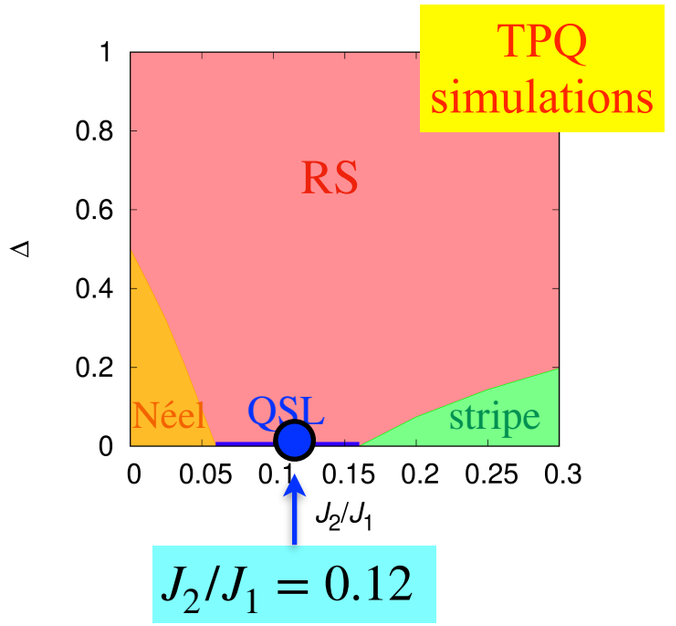




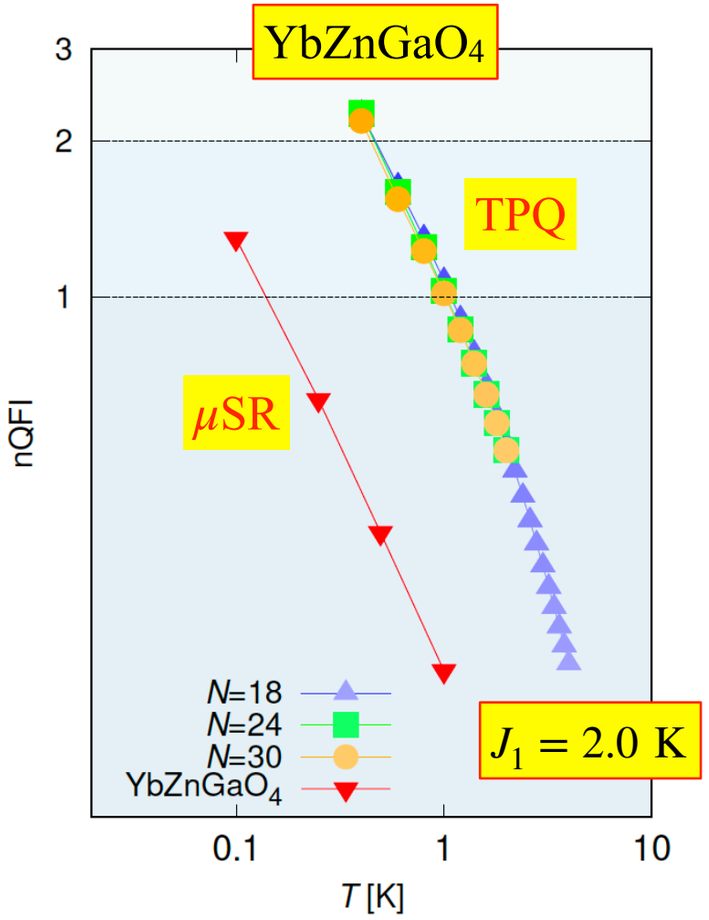
simulations of $KYbSe_2$ reproduce power-law scaling, with similar exponent to experiment, but higher absolute value of QFI

T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874

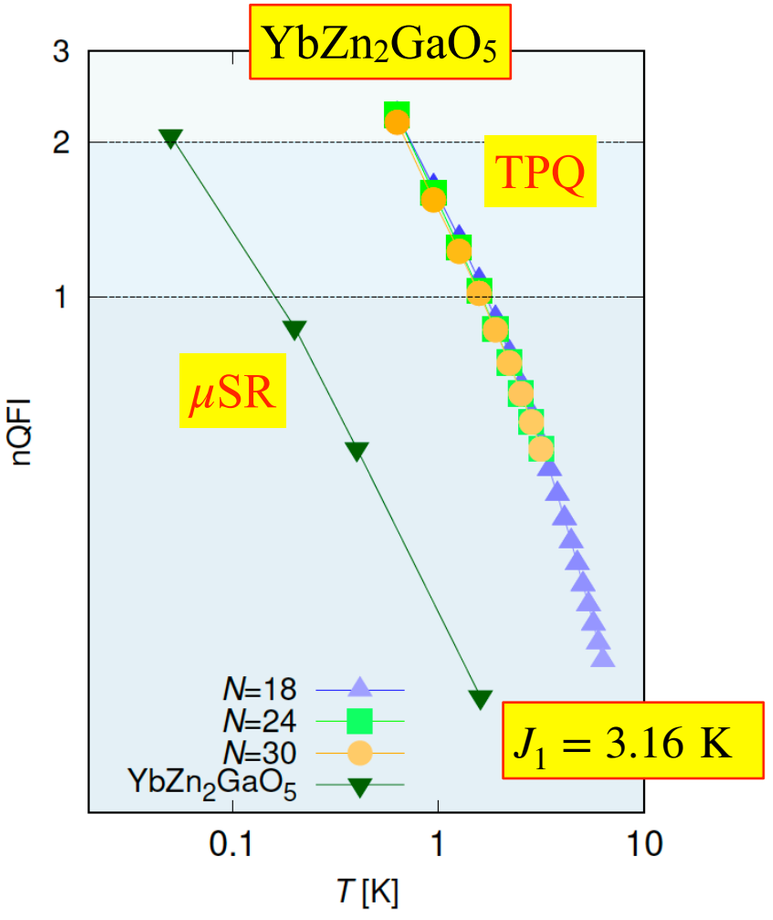
what about μMSR results for QSL candidates ?



simulations of $YbZnGaO_4$ and $YbZn_2GaO_5$ reproduce power-law scaling, with similar exponent to experiment, but higher absolute value of QFI

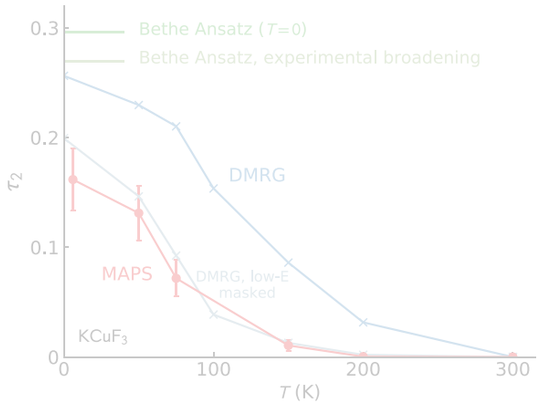


F. L. Pratt *et al,*
Phys. Rev. B **106**, L060401 (2022).



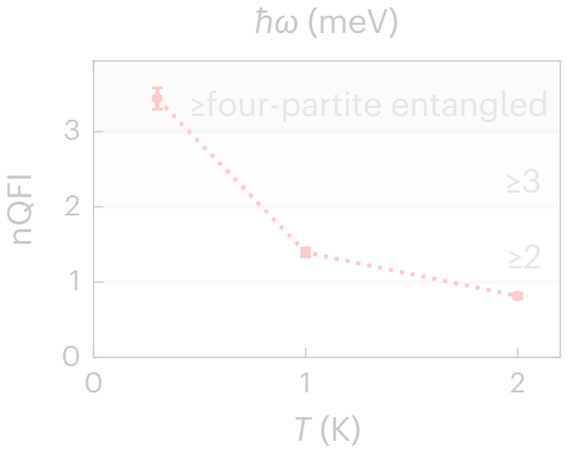
Hank C. H. Wu *et al.*,
arXiv:2502.00130.

what do we need to understand ?



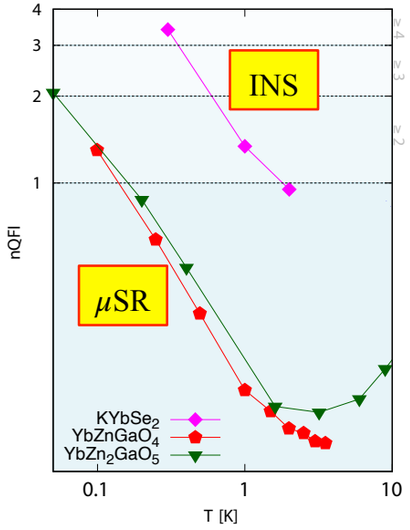
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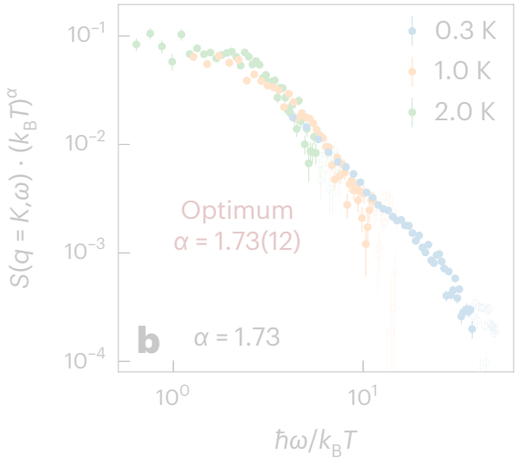
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how should we interpret the power-law scaling of QFI seen in triangular lattice AF's ?

characteristic of low-T growth of entanglement in QSL and Néel phases

bonus question



can we understand the way in which fluctuations scale with energy and temperature in the triangular lattice AF $KYbSe_2$?

where does this leave us ?

where might the quantum information approach be useful ?





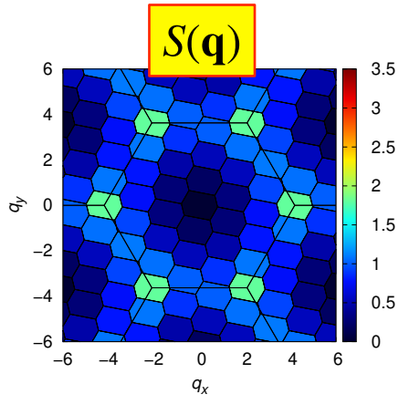
distinguishing phases with qualitatively similar equal-time correlations, but qualitatively different entanglement properties...

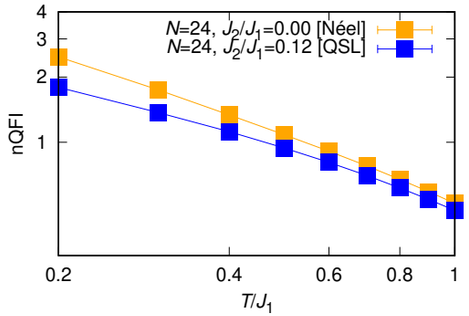


does this idea work ?



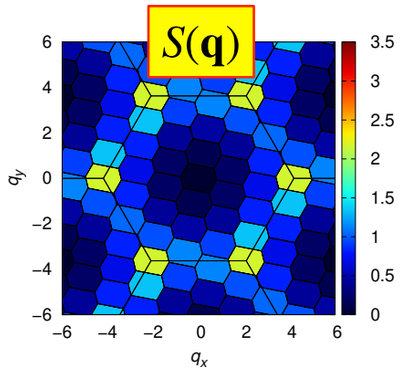
tools taken from quantum information can be helpful in (re)interpreting the results of inelastic neuton scattering



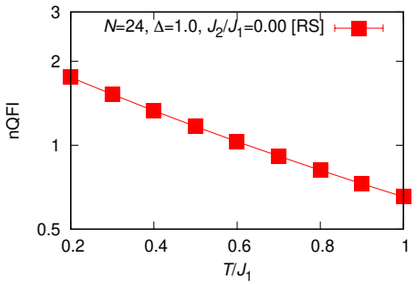


QSL

$$f_{\mathcal{Q}}[\mathbf{q}_{\mathsf{K}}, T/J_1] \propto b(T/J_1)^{-c}$$



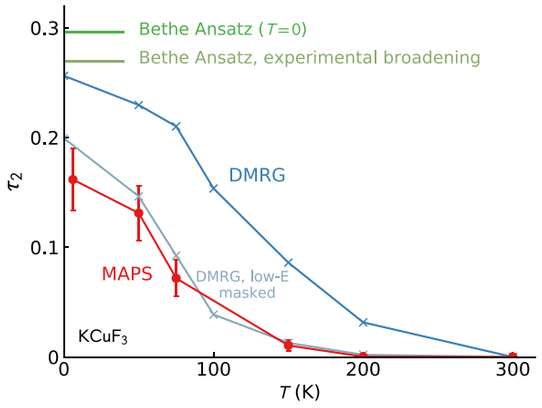
RS



$$f_{\mathcal{Q}}[\mathbf{q}_{\mathsf{K}}, T/J_1] \propto b' e^{-c'(T/J_1)}$$

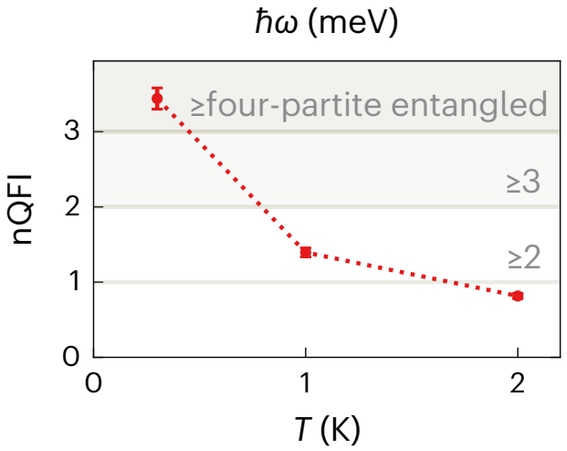
in some cases, quantum Fisher information (QFI) provides a means of distinguishing phases with qualitatively similar equal-time correlations, but qualitatively different entanglement properties

what else have we learned ?



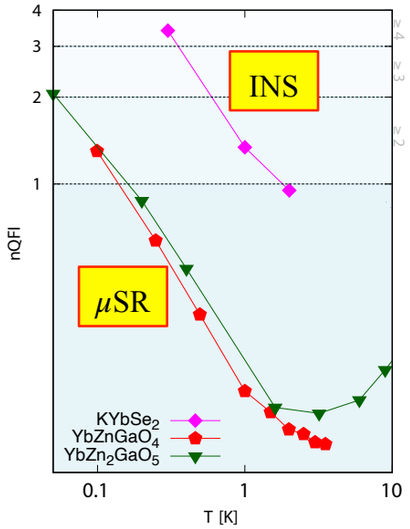
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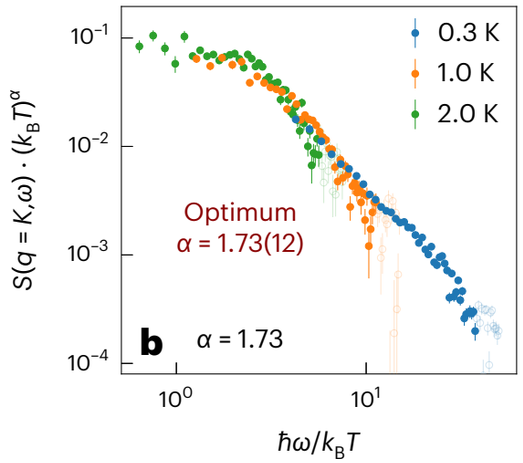
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characteristic of low-T growth of entanglement in QSL and Néel phases

bonus question



can we understand the way in which fluctuations scale with energy and temperature in the triangular lattice AF $KYbSe_2$?

feel free to ask !

thanks for listening !

1D results

PHYSICAL REVIEW RESEARCH **7**, 023271 (2025)

Witnessing disorder in quantum magnets

Snigdh Sabharwal,* Tokuro Shimokawa, and Nic Shannon
Theory of Quantum Matter Unit, Okinawa Institute of Science and Technology Graduate University, Onna-son, Okinawa 904-0412, Japan

(Received 30 July 2024; accepted 12 May 2025; published 17 June 2025)

2D results

Can experimentally-accessible measures of entanglement distinguish quantum spin liquids from disorder–driven "random singlet" phases ?

Tokuro Shimokawa,[1,*] Snigdh Sabharwal,[1] and Nic Shannon[1]
[1]*Theory of Quantum Matter Unit, Okinawa Institute of Science and Technology Graduate University, Onna-son, Okinawa 904-0412, Japan*
(Dated: May 30, 2025)

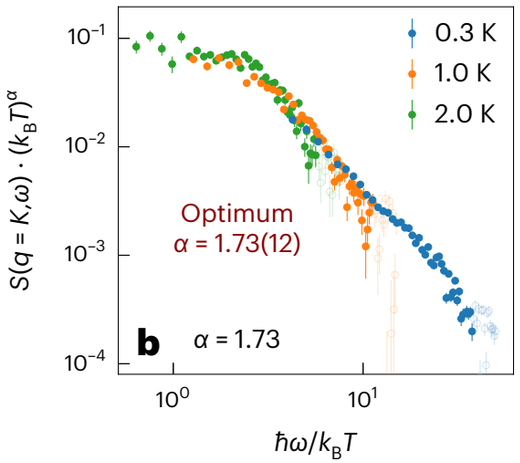
https://arxiv.org/abs/2505.11874

OIST's visiting program: https://www.oist.jp/visiting-program/application-visiting-scholars

postdoc positions: https://www.oist.jp/careers/postdoctoral-scholars-theory-quantum-matter-unit

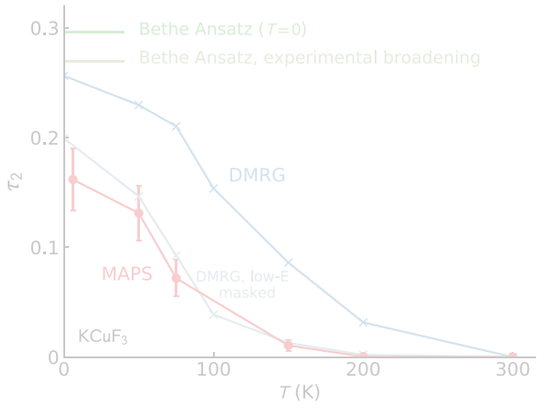
bonus question...

if there's time...



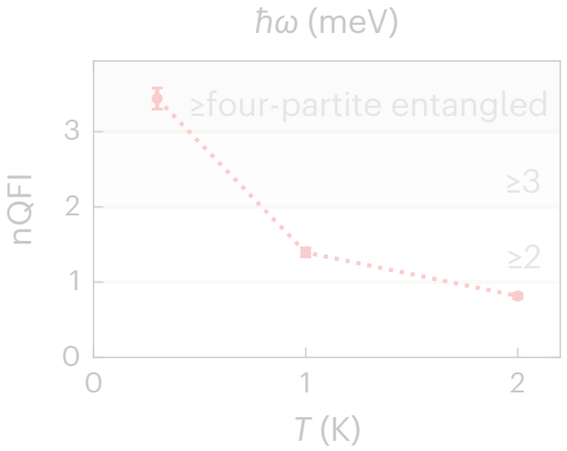
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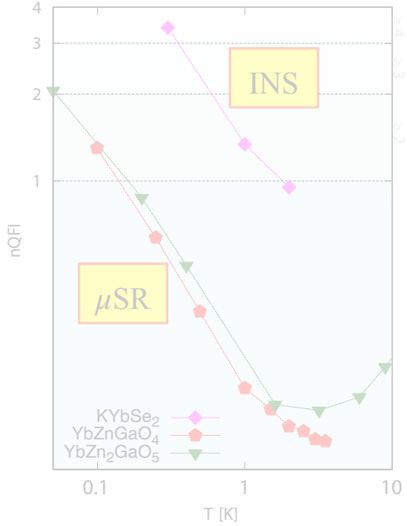
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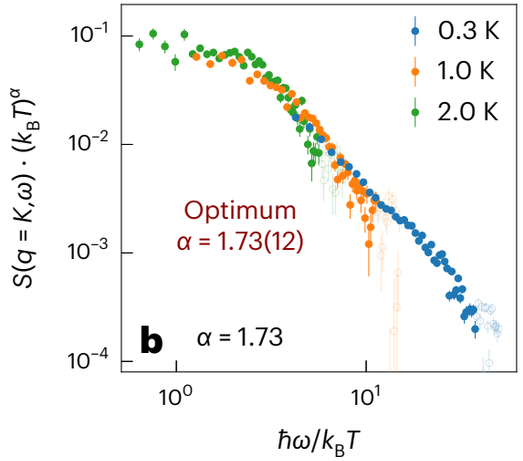
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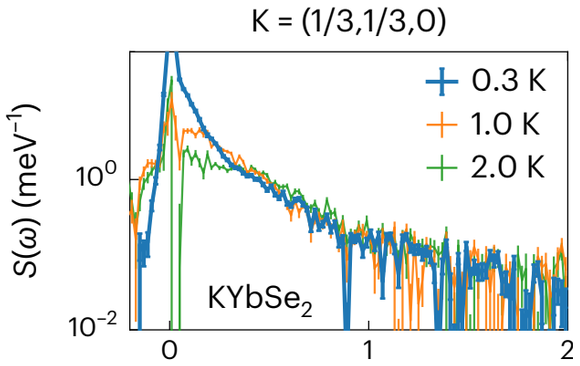
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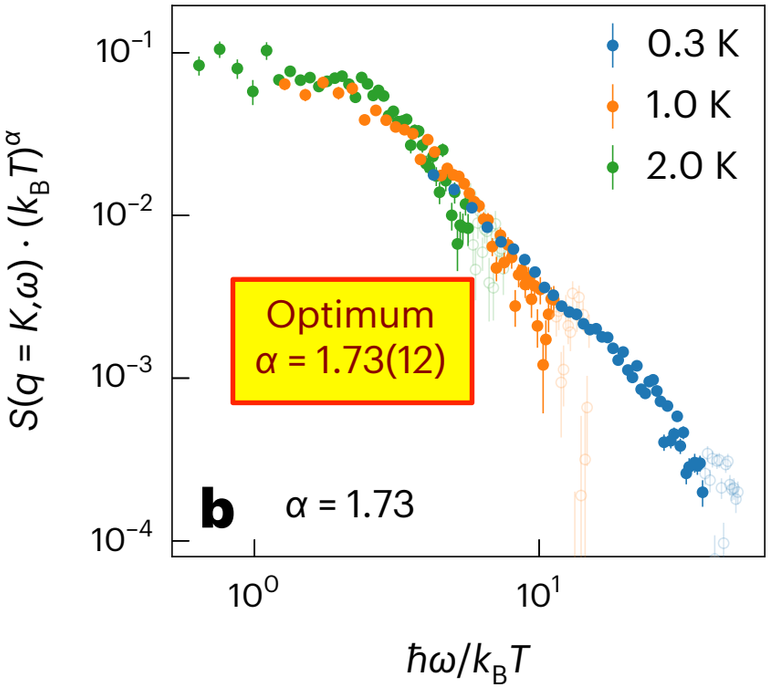
bonus question

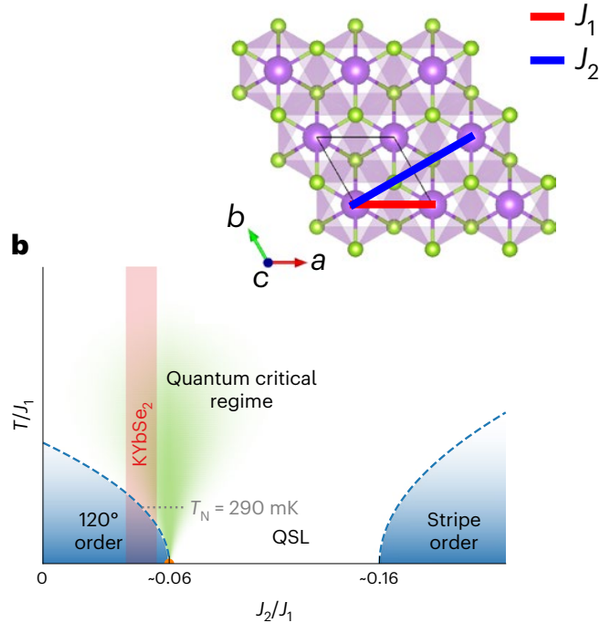


can we understand the way in which fluctuations scale with energy and temperature in the triangular lattice AF $KYbSe_2$?

what do fluctuations in $KYbSe_2$ reveal ?







continuum
of excitations
at ordering vector

scaling collapse as function $\hbar\omega/k_BT$
⇒ nearby quantum critical point (QCP) ?

interpreted in terms of J_1-J_2
Heisenberg model with
$J_2/J_1 \approx 0.047$

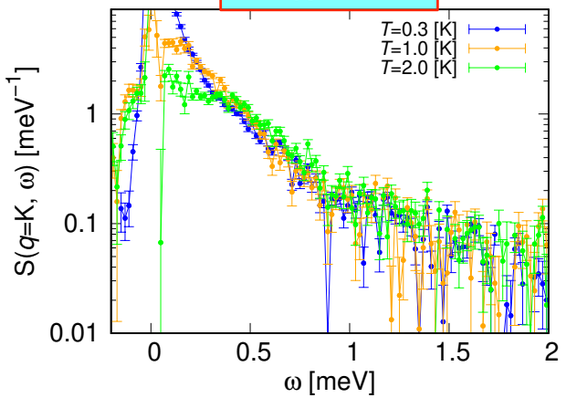
A. Scheie *et al.*, Nat. Phys. **20**, 74 (2024)



does simulation also reveal a scaling collapse ?

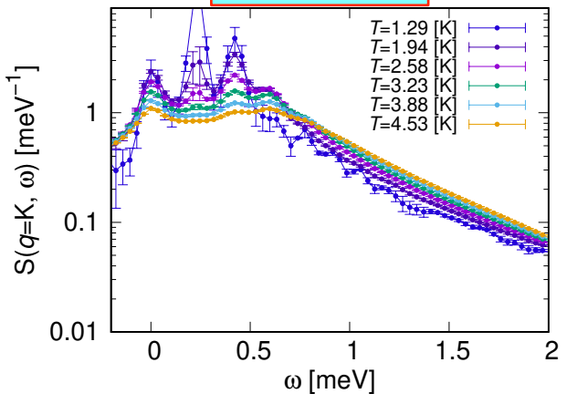
KYbSe$_2$

$S(\mathbf{q}_K, \omega, T)$

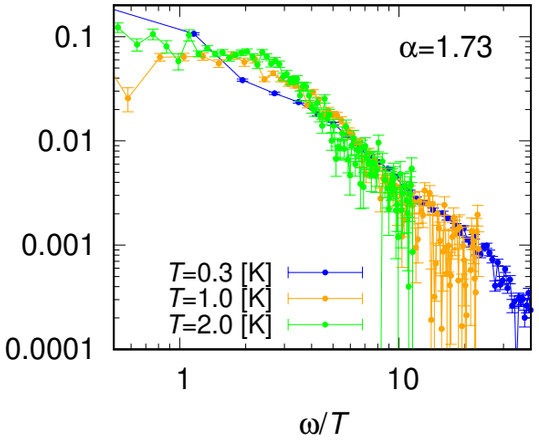


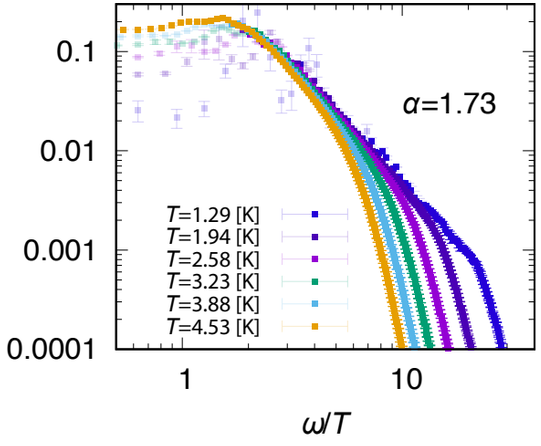
J_1-J_2 model

$S(\mathbf{q}_K, \omega, T)$

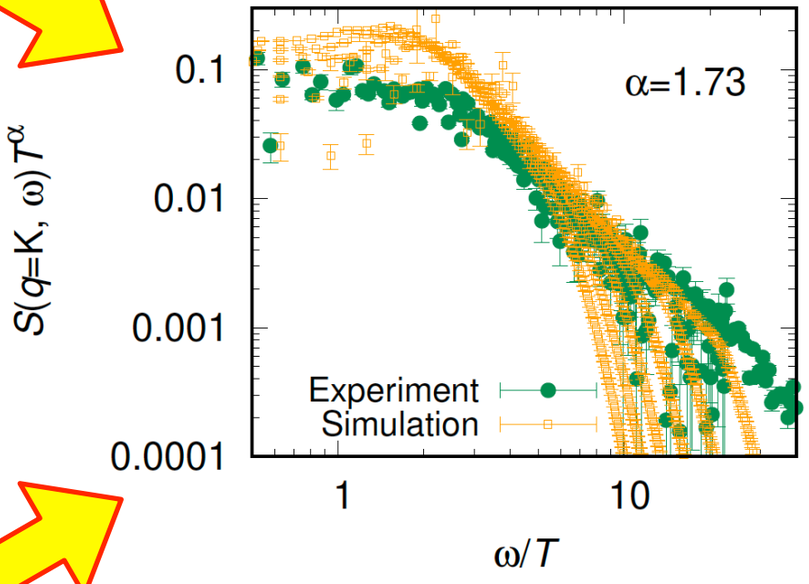


scaling collapse, $\alpha = 1.73$





reasonably good agreement between scaling in theory and experiment using model, parameters and exponent proposed for KYbSe$_2$



T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874

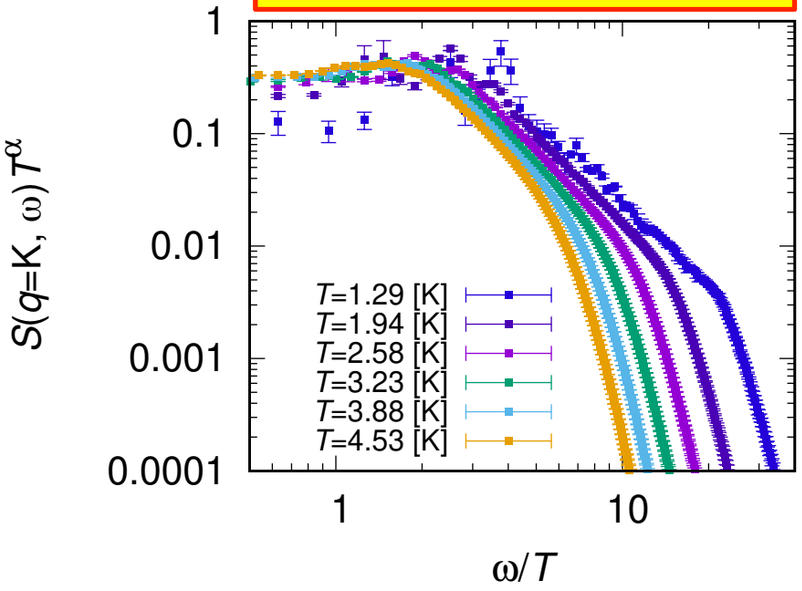
was this the best choice of exponent ?

TPQ calculations
for $N = 30$ spins

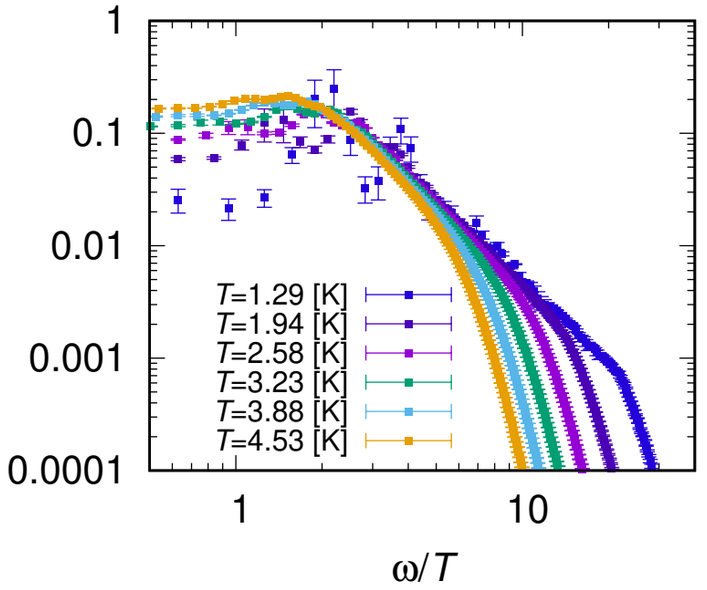
$$\mathscr{H} = J_1 \sum_{\langle ij \rangle_1} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle ij \rangle_2} \mathbf{S}_i \cdot \mathbf{S}_j$$

$J_2/J_1 = 0.047$

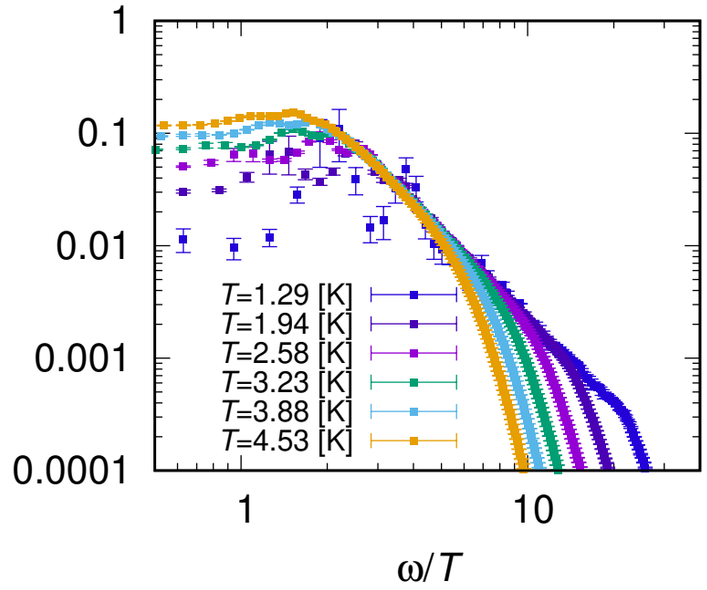
scaling collapse, $\alpha = 1.0$



scaling collapse, $\alpha = 1.73$



scaling collapse, $\alpha = 2.1$



marginally better collapse for $\alpha = 2.1$, relative to $\alpha = 1.73$, but several decades of ω/T needed to give definitive answer

T. Shimokawa, S. Sabharwal and N. Shannon, arXiv:2505.11874



where have we seen this kind of thing before ?



PHYSICAL REVIEW B VOLUME 49, NUMBER 17 1 MAY 1994-I

Theory of two-dimensional quantum Heisenberg antiferromagnets with a nearly critical ground state

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Quantum phase transitions in frustrated quantum antiferromagnets

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quantum critical scaling form for $S(k,\omega)$

$$S(k,\omega) = \frac{2\pi\hbar N_0^2}{N\rho_\perp}\left(\frac{Nk_BT}{4\pi\rho_\perp}\right)^{\bar\eta}\left(\frac{\hbar c_\perp}{k_BT}\right)^2 \frac{2}{1-e^{-\bar\omega}}\Xi_1\left(\bar k, \bar\omega, x_1, y_\rho, y_\chi\right). \qquad \bar\eta = 1 + \frac{32}{3\pi^2 N};$$

$\bar\omega = \frac{\hbar\omega}{k_BT}$

inference that $KYbSe_2$ lies close to a QCP seems reasonable

thanks for listening !

1D results

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Witnessing disorder in quantum magnets

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2D results

Can experimentally-accessible measures of entanglement distinguish quantum spin liquids from disorder–driven "random singlet" phases ?

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https://arxiv.org/abs/2505.11874

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